Differential Equations (MAT256)

Mid-semester Examinations **(Set 2)** 19th September, 2022
F.M.: 50 Time: 1 hour 55 minutes

1. Solve the differential equation $\frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3}$. **(9)**

2. Solve the equation $(x+1)\frac{dy}{dx} + (x+2)y = 2\,x\,e^{-x}$ and give the largest interval I over which the general solution is defined. **(Using any formula will not be acceptable. You must derive all steps.) (9)**

3. (a) Draw the direction field for $\frac{dy}{dx} = 1 - xy$. (**Mark special lines, if any, on the diagram.)**

   (b) On the direction field, draw the solution curve that passes through the point $y(0) = 0$. **(The reason for drawing a particular solution curve should be evident from your diagram.) (15)**

4. (a) Find the critical points and phase portrait of the differential equation $\frac{dy}{dx} = y^2 - y^3$.

   (b) Classify each critical point as stable, unstable and semi-stable.

   (c) Sketch typical solution curves in the regions in the xy-plane determined by the graphs of the equilibrium solutions. **(10)**

5. Show that $y = e^{cosx} \int_0^x dt\; t\; e^{-cost}$ is a solution of the differential equation $\frac{dy}{dx} + (sin\, x)y = x$. **(You need to show the details of the differentiation; stating the final result/or reference to theorems without proof will not be acceptable.) (7)**