Ahmedabad University

School of Arts and Sciences

PHY312 - Quantum Mechanics - 2

BS programme

End Semester Examination, Winter 2024

Total marks : 40 **Time : Two hours**

Enrolment Number: .....

Date: April 22, 2024 (05:00 AM TO 07:00 PM)

No. of printed pages: 02

---

1. (8 marks) A hydrogen atom is placed in a (time-dependent) electric field $\vec{E} = E(t)\hat{z}$. Calculate all four matrix elements H'_{ij} of the perturbation $H' = eEz$ between the ground state ($n = 1$) and the (quadruply degenerate) first excited states ($n = 2$).

2. (2 marks) The half-life $t_{1/2}$ of an excited state is the time it would take for half the atoms in a large sample to make a transition. Find the relation between $t_{1/2}$ and τ (the "lifetime" of the state).

3. (10 marks) For a harmonic oscillator, let $|n\rangle$ be a normalised eigenvector of the number operator (N) with eigenvalue n. In the basis of eigenvectors of N,

   (a) (2 marks) Find the expression for the matrix elements of annihilation and creator operators

   (b) (2 marks) Find the expression for the matrix elements of position operator

   (c) (1 marks) Find the matrix representation of annihilation and creator operators

   (d) (1 marks) Find the matrix representation of position operator.

   (e) (4 marks) Find the position operator in the Heisenberg picture at time t.

4. (10 marks) Fig (1) shows a schematic diagram of Stern-Gerlach experimental set up as well as the result.

   (a) (1.5 marks) Using the labels in the figure, explain why classically, we should observe only one large spot centered in (dashed lines in the figure on the right).

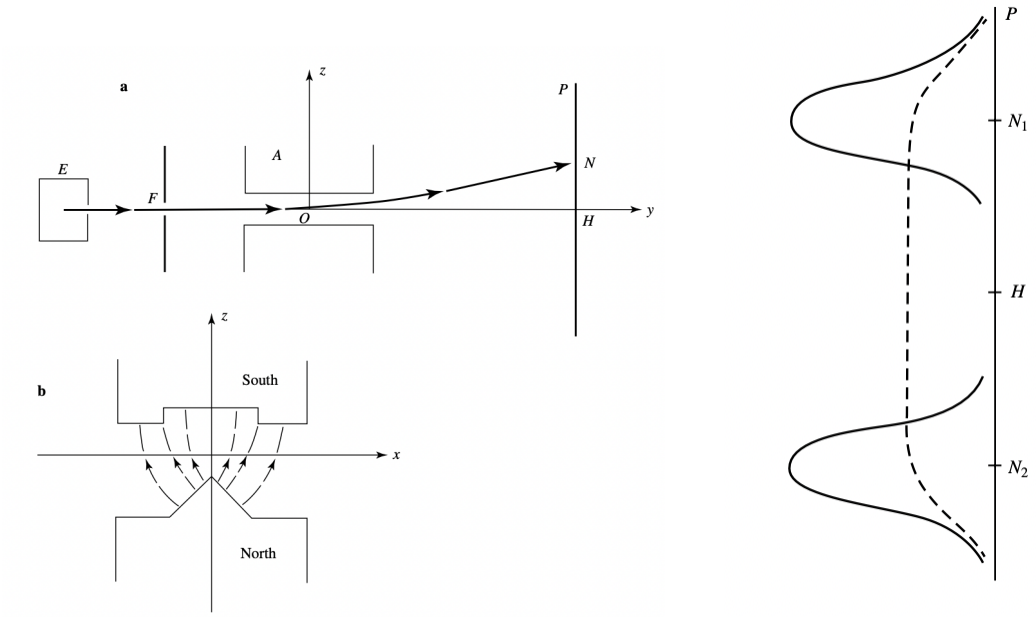
   (b) (1.5 marks) How does QM explain the actual result: two spots, centered at N_1 and N_2, on the screen.

   (c) (1 mark) With justification, state whether the following statement is true or false: "*When we pierce a hole in the plate at the position of the spot N_1, the atoms that pass through this hole are all in the spin state $|+\rangle$*".

   (d) (2.5 marks) Recall that, the theory of angular momentum tells us that if $\hat{S}_x, \hat{S}_y$ and $\hat{S}_z$ satisfy the angular momentum commutation relations, then

   $$\hat{S}_\pm |s\ m\rangle = \hbar\sqrt{s(s+1) - m(m \pm 1)}|s\ (m \pm 1)\rangle\ ,$$

   all the symbols have the usual meaning. Using the eigenvectors of S_z operator as basis set, arrive at the matrix representation of S_z, S_x and S_y operators.



(a) Set up (b) Result

Figure 1: Stern Gerlach experimental set up (**a** shows trajectory of a sliver atom emitted from the furnace E and **b** shows a cross section in the plane of the electromagnet.) and result (spots observed on the plate in the Stern-Gerlach experiment: dashed line according to CM and two spots according to QM.)

(e) (2 marks) Express eigenvectors of S_x and S_y operators in terms of eigenvectors of S_z operator.

(f) (1.5 marks) Suppose, using SG experimental set up, we wish to prepare a beam of silver atoms which are all in an eigenstate of S_x operator, what shall we do?

5. (10 marks) Addition of angular momenta: Consider the spin state space of two spin-half particles

(a) If $\vec{S}_1$ and $\vec{S}_2$ are angular momenta, show that, $\vec{S} = \vec{S}_1 + \vec{S}_2$ is also angular momentum.

(b) Show that $\vec{S}_1 \cdot \vec{S}_1 = \frac{1}{2}(S_{1+}S_{2-} + S_{1-}S_{2+}) + S_{1z}S_{2z}$, hence, show that $\vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 + S_{1+}S_{2-} + S_{1-}S_{2+} + 2S_{1z}S_{2z}$.

(c) Let $\{\vec{S}_1^2, \vec{S}_2^2, S_{1z}, S_{2z}\}$ be CSCO-1 and $\{\vec{S}_1^2, \vec{S}_2^2, \vec{S}^2, S_z\}$ be CSCO-2. This

$$\{|+,+\rangle, |+,-\rangle, |-,+\rangle, |-,-\rangle\} \tag{1}$$

is a basis of common eigenvectors of CSCO-1. Show that $\frac{|+,-\rangle + |-,+\rangle}{\sqrt{2}}$ is an eigenvector of each operator in CSCO-2 and find the corresponding eigenvalues.