Enrolment No. _____

AHMEDABAD UNIVERSITY

SCHOOL OF ARTS AND SCIENCES

UNDERGRADUATE PROGRAMMES

2025-2026 WINTER SEMESTER

MID-SEMESTER EXAMINATION

MAT142 INTRODUCTORY CALCULUS

Date: 01-03-2026 Total Marks: 50

Time: 02.00 pm – 4.00 pm No. of printed pages: 16

Name of the Student: _____

Enrolment No.:_____ Section No.: _____

Question No.	1	2	3	4	5	6	7	8	Total Marks Obtained
Marks									

Examiner's Signature: _____ Invigilator's signature: _____

General Instructions:

1. Students must carry their identity card during the examination.
2. No break is allowed during the examination under any circumstances. In case of an emergency, with prior permission of the invigilator and an escort, a break of a maximum of five minutes is allowed.
3. No unauthorized devices, such as mobile phones, any kind of watches, other gadgets, or any kind of material, are allowed on a person.
4. Any violation of examination rules and an intent of malpractice will lead to strict disciplinary action, including a possible expulsion from the university.

Course Specific Instructions:

1. Each question is compulsory.
2. Use the given space to write the answers.
3. If more space is needed, the supplementary can be provided on request.
4. The use of a simple calculator is allowed.

Question 1 (10 Marks)

Riya Mehta's venture, Smart Tiffin, quickly gained popularity among busy professionals across Ahmedabad's major corporate zones, including Prahlad Nagar and SG Highway. What began as a modest lunch delivery service from a small rented kitchen was now handling thousands of meal orders each month. Encouraged by rising demand, Riya started pushing for higher sales volumes through corporate tie-ups and referral discounts. However, her operations manager raised a concern during the monthly review meeting. While revenues were steadily increasing, the kitchen team was facing increasing coordination pressure. Delivery partners were demanding overtime pay during peak hours, and the kitchen workflow was becoming increasingly congested.

After carefully studying three months of operational data, the analytics team concluded that Smart Tiffin's monthly cost structure was no longer purely linear. Instead, the total cost (in ₹) of producing and delivering xxx lunch boxes could be reasonably modelled as:

$$C(x) = 1{,}20{,}000 + 40x + 0.02x^2$$

The fixed component reflects rent, salaries, and platform maintenance. The linear term captures normal ingredient and delivery costs, while the quadratic term represents rising inefficiencies at higher production levels.

The selling price has remained stable at ₹120 per lunch box. Riya now wants a careful profitability assessment before committing further marketing funds. She has asked you, as the business analyst, to evaluate the situation.

(a) Write the profit function for Smart Tiffin.

(b) Find the profit when 2,000 lunch boxes are sold.

(c) Determine the break-even level(s) of sales.

(d) Examine whether the profit function is even, odd, or neither. Justify mathematically.

(e) Due to competitive pressure, suppose Smart Tiffin reduces the selling price from ₹120 to ₹110 per lunch box. Determine the new break-even sales level(s) and briefly interpret the impact of the price reduction on profitability.

Question 2 (04 Marks)

A start-up in Ahmedabad called ZoomRide rents electric scooters to tourists visiting the Sabarmati Riverfront. The management noticed that as more scooters are rented, daily revenue increases, but not always at a constant rate, due to discount bundles.

Based on recent data, the daily revenue (in ₹) from renting x scooters is modeled by:

$$R(x) = 500x - 0.1x^2$$

where x is the number of scooters rented in a day.

The operations manager wants to understand how quickly revenue changes across a range of demand levels. Find the average rate of change of revenue when the number of scooters rented increases from 40 to 60**.** Interpret the result in business terms. Interpret the result from a managerial perspective.

Question 3

(a) (04 Marks)

A fast-growing tech star-tup in Ahmedabad purchases cloud storage in bulk. The finance team has observed that as the storage volume increases, the average cost per GB gradually stabilizes because fixed overhead gets spread over more data and vendors offer bulk discounts.

After analysis, the average cost per GB (in ₹) when the company purchases xxx terabytes of storage is modelled as:

$$C(x) = \frac{3x^2 + 500x + 20{,}000}{x^2 + 40x}$$

The CTO wants to understand the long-run average cost behaviour before committing to very large storage contracts. As the amount of storage purchased becomes very large, what will be the average cost per GB in the long run? Interpret the result from a managerial perspective.

Question 3

(b) (04 Marks)

Quick-Bite, a food delivery start-up operating in Ahmedabad, is testing a new routing algorithm for its delivery partners. The analytics team models the additional delivery time per order (in minutes) due to route adjustments when the traffic disruption level is very low.

The additional delivery time is estimated by:

$$T(x) = \frac{6x^2 + 15x}{x}$$

where x represents the traffic disruption index (a small positive number).

The operations manager wants to understand how the delivery delay behaves as traffic disruption becomes negligible.

Question 4

(a) (04 Marks)

Urban-Basket, a fast-growing grocery delivery platform in Ahmedabad, uses a congestion index xxx to predict expected delivery time during peak hours. The analytics team wants the delivery time function to behave smoothly when congestion reaches the critical level $x = 2$

Recently, customer feedback revealed that even small inconsistencies in promised delivery time were leading to confusion at checkout. The product team is therefore conducting a careful audit of the mathematical model before deploying it in the live pricing engine. They have particularly flagged the threshold $x = 2$ as operationally sensitive during evening peak demand.

After internal modelling, the expected delivery time (in minutes) is given by:

$$T(x) = \begin{cases} \dfrac{4x^2 + 5x - 6}{x + 2}, & x < 2 \\ kx - 1, & x = 2 \\ 3x - 1, & x > 2 \end{cases}$$

Management wants to choose k so that customers do not notice a sudden jump in the promised delivery time at the threshold.

Question 4
(b) (04 Marks)

Fresh-Sip, a rapidly expanding smoothie chain in Ahmedabad, is fine-tuning its demand-based price multiplier for peak hours. After deeper analytics, the team proposes that when demand is in the range $0 < x < 4$, the multiplier should follow

$$F(x) = \frac{x^2 + 5x - 6}{x + 2}$$

while for demand greater than 4, it should follow

$$F(x) = 2x - 2$$

To prevent any sudden jump in pricing, management fixes the multiplier at exactly $x = 4$ to be 6. Before implementing the policy, the analytics team wants to verify that the pricing rule behaves smoothly at the critical demand level. Using the given information, check whether the resulting function is continuous at $x = 4$ and briefly interpret the result from a managerial perspective.

Question 5 (04 Marks)

Fresh-Cart, an online grocery delivery platform in Ahmedabad, monitors the late delivery penalty (in ₹) near a critical congestion index of $x = 5$. Because different routing rules apply just before and after this threshold, analysts model the penalty as follows: when congestion is below 5,

$$P(x) = \frac{x^2 - 25}{x - 5}$$

and when congestion is above 5,

$$P(x) = \frac{\sqrt{x + 20} - 5}{x - 5}$$

Management wants to know whether the penalty behaves smoothly near the critical level. Evaluate the left-hand and right-hand limits of $P(x)\ as\ x \to 5$ and determine whether the two-sided limit exists. Briefly interpret the result from an operational perspective.

Question 6 (04 Marks)

Quick-Charge, an electric vehicle charging network in Ahmedabad, monitors the average power adjustment factor $A(x)$ near the standard load level $x = 1$. Engineers model the adjustment factor as

$$A(x) = \frac{x^2 + x - 2}{x - 1}$$

Because the function is not defined exactly at $x = 1$, the operations team decides to estimate the value numerically using nearby load values.

Using the tabular method, estimate $\lim_{x \to 1} A(x)$ and interpret the result in operational terms.

Question 7 (04 Marks)

Swift-Net, a cloud services provider in Ahmedabad, monitors how the average server response time (in milliseconds) changes as the number of simultaneous users x increases. Based on performance data, engineers model the response time as

$$R(x) = \frac{200}{x - 50} + 5,$$

where $x > 50$

The IT team wants to understand the long-run behaviour of the response time and identify any asymptotes in the model.

Find the vertical and horizontal asymptotes of $R(x)$. Briefly interpret their meanings in the context of server performance.

Question 8 (08 Marks)

Evaluate the limits of the following:

(i)

$$\lim_{x \to 2} \frac{\sqrt{x+7}-3}{x-2}$$

(ii)

$$\lim_{x \to 0} \frac{\sqrt{9 + 2x} - 3}{x}$$

(iii)

$$\lim_{x\to\infty} \sqrt{x^2 + 6x} - x$$

(iv)

$$\lim_{x\to -4} \frac{x^2 + 7x + 12}{x + 4}$$

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