Ahmedabad University

School of Arts and Sciences

MAT248 Applied Linear Algebra

Mid Term Examination 2021-22 (Winter Semester)

Total Marks = 100 **Time - 2 hours**

Name & Enrolment Number:
Date:
No. of printed pages: 3

1. (10 points) Find the quadratic polynomial whose graph goes through the points $(-2, 13)$, $(0, 3)$, and $(1, 10)$.

2. (10 points) Consider a linear system whose augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & 5 & -2 \\ 1 & 2 & -4 & -1 \\ 2 & 6 & k & 1 \end{array}\right].$$

   For what value of k will the system have no solutions?

3. (5 points) Determine how many pivots each of the following matrices have.

   a. $\begin{bmatrix} 1 & 0 & -9 & 6 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 9 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & -7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 10 \end{bmatrix}$

4. (5 points) The reduced row-echelon forms of the augmented matrices of four systems are given below. How many solutions does each system have?

a. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 18 \\ 0 & 0 & 1 & -10 \end{array}\right]$

b. $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

c. $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -16 \\ 0 & 0 & 1 & 8 \end{array}\right]$

d. $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

5. (10 points) Let

$$A = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 2 & 0 & 0 & -10 \end{bmatrix}.$$

Describe all solutions of $A\vec{x} = \vec{0}$.

6. (10 points) Find all k such that vectors

$$\vec{u} = \begin{bmatrix} -4 \\ -9 \\ 26 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 3 \\ 9 \\ -36 + k \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$$

are linearly dependent.

7. (10 points) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix},$$

then find $T\left(\begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}\right)$.

8. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation that first projects points onto the yz-plane and then reflects around the line $y = -z$.

   (a) (6 points) Find the standard matrix A for T.

   (b) (4 points) Is T invertible? Justify your answer.

9. (10 points) Let $\mathcal{P}_n$ be the vector space of all polynomials of degree n or less in the variable x. Let $D : \mathcal{P}_3 \to \mathcal{P}_2$ be the linear transformation defined by $D(p(x)) = p'(x)$. That is, D is the derivative operator. Let

$$\begin{aligned} \mathcal{B} &= \{1, x, x^2, x^3\}, \\ \mathcal{C} &= \{1, x, x^2\}, \end{aligned}$$

   be ordered bases for $\mathcal{P}_3$ and $\mathcal{P}_2$, respectively. Find the matrix $[D]_{\mathcal{B}}^{\mathcal{C}}$ for D relative to the basis $\mathcal{B}$ in the domain and $\mathcal{C}$ in the codomain.

10. (10 points) Let

$$A = \begin{bmatrix} -1 & -3 & -4 & 0 \\ 3 & -2 & -2 & 1 \\ 4 & -10 & -12 & 2 \end{bmatrix}$$

    be the matrix associated to the transformation T.

    (a) (2 points) Is T one-to-one? Justify your answer.

    (b) (2 points) Is T on-to? Justify your answer.

    (c) (2 points) Is T invertible?

    (d) (4 points) Find the inverse of the following matrix.

$$B = \begin{bmatrix} -1 & -3 \\ 3 & -2 \end{bmatrix}.$$

11. (10 points) Either prove the following statement or give a counter example. If a 2×2 matrix A is such that $A^2 = 0$, then $A = 0$.

End of Exam.
