Ahmedabad University

School of Arts and Sciences

MAT 281 Multivariable Calculus

Mid Term Examination 2024 (Bi-Semester)

Total Marks = 30 Time – 2 hours

Instructions:

- Each question is compulsory.
- Calculator is not allowed.
- Handwritten notes and the text-book are allowed.
- Use of any electronic device is strictly forbidden.

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1. Evaluate
$$\lim_{n\to\infty}\left[\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\cdots+\frac{1}{(n+n)^2}\right].$$
Jusitify. (Marks: 4)

2. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function defined as follows:
$$f(x,y)=\begin{cases}(x^2+y^2)\sin\frac{1}{\sqrt{x^2+y^2}}, & \text{if } x^2+y^2\neq 0,\\ 0, & \text{if } x^2+y^2=0.\end{cases}$$
Show that the function is differentiable at $(0,0)$. (Marks 4)

3. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function defined as follows:

$$f(x,y) = \begin{cases} (x^3 \sin \frac{1}{x^2} + y^3 \sin \frac{1}{y^2}, & \text{if } xy \neq 0, \\ 0, & \text{if } xy = 0. \end{cases}$$

Is differentiable at $(0,0)$. (Marks 4)

4. Find z_x and z_y for the function $z = x^{\log(x^y)}$. (Marks 4)

5. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function defined as follows:

$$f_\alpha(x,y) = \begin{cases} \frac{xy}{(x^2+y^2)^\alpha}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

For which values of α, f_α is continuous at the point $(0,0)$. (Marks 6)

6. Evaluate $\lim_{(x,y)\to(0,0)} \frac{\sqrt{4-x+y}-2}{7x-7y}$. (Marks 3)

7. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function defined as follows:

$$f_\alpha(x,y) = \begin{cases} \frac{(x-y)^3}{(x^2+y^2)}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Does the directional derivative along the direction $(1,-1)$ at the point $(0,0)$ exist? (Marks 5)