Ahmedabad University

School of Arts and Sciences

MAT142 INTRODUCTORY CALCULUS

End Semester Examination 2024 (Monsoon Semester)

Total Marks = 60 Marks **Time: 2:00 pm to 4:30 pm**

Date: 02-12-2024 **No. of printed pages: 16**

Name of the candidate		Section No.
Enrolment No.		

Examiner's Signature: _____ Invigilator's signature: _____

Que. No.	1	2	3	4	5	6	7	8	9	10	11	12	Total Marks
Marks													

Instructions:

1. Each question is compulsory and carries 5 marks.
2. The use of a simple calculator is allowed.
3. Use the given space to write the answers. If more space is needed, supplementary will be provided upon request.

Q 1: (a) The marginal cost function of a commodity is given

$$MC = \frac{14000}{\sqrt{7x+4}}$$

and the fixed cost is ₹18,000. Find the total cost.

Q 1: (b) Evaluate:

$$\int_{100}^{100} \frac{t^{-1/2}(t^2 - t + 2)\left(3e^{-5t} + \sqrt{t}\right)(4t^2 - 3)^5}{\sqrt{t}(t^2 - 1)\left(3t^2 - \sqrt{5t} + 2\right)(t + 1)(t^2 + 2t + 5)^{12}}\, dt$$

Q 2: (a) A manufacturer determines that when x thousand units of a particular commodity are produced, the profit generated will be

$$P(x) = -400x^2 + 6{,}800x - 12{,}000$$

dollars. What will be the marginal profit when 9,000 units are produced? Interpret your findings.

Q 2: (b) Find the first-order and second-order derivatives of the following functions:

$$f(x) = \frac{3x^2 + 2}{5x - 1}$$

Q 3: A business manager determines the cost of producing x units of a particular commodity is C thousands of dollars, where $C(x) = 0.04x^2 + 5.1x + 40$.

a) Find the average rate of change in cost as the level of production changes from $x = 10$ to $x = 11$ units.

b) Use calculus to find the instantaneous rate of change in cost with respect to production level when $x = 10$. Is the cost increasing or decreasing when $10\ units$ are being produced?

Q 4: It is estimated that x months from now, the population of a certain community will be

$P(x) = x^2 + 20x + 8{,}000.$

a) At what rate will the population be changing with respect to time 15 months from now?

b) By how much will the population actually change during the 16^{Th} month?

Q 5: (a) The cost function of a particular commodity is

$$C(x) = 48400 + 200x + x^2$$

Find the average cost function and determine the production level x that will minimize the average cost. Also, find the minimum average cost.

Q 5: (b) Find the first-order and second-order derivatives of the following function:

$$y = -\frac{x^2}{16} + \frac{2}{x} - x^3 + \frac{1}{3x^2} + \frac{x}{3}$$

Q 6: (a) Find the first-order derivative of the following functions:

$f(x) = (3x + 1)^4 (2x - 1)^5$

Q 6: (b) Find the first-order derivative of the following functions:

$f\ (x) = \log\ (x^{15}15^{x}e^{e})$

Q 7: (a) A tank contains 5000 liters of pure water. Brine (very salty water), which contains 30 grams of salt per liter of water, is pumped into the tank at a rate of 25 liters per minute. The concentration of salt water after t minutes (in grams per liter) is

$$C(t) = \frac{30t}{200 + t}$$

What happens to the concentration as $t \to \infty$?

Q 7: (b) Evaluate:

$$\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5}$$

Q 8: (a) A certain manufacturer makes a line of luxurious chairs. It is estimated that the total cost of making x luxurious chairs is $C(x) = 350x + 200{,}000$ dollars per year. What will the average cost be of making x chairs in the long run?

Q 8: (b) The ratio of revenue to cost from total sales of a Pulitzer-prized book is approximated by the function

$$T(x) = \frac{100x + 120x^3 - x^4}{2x^2 + 300x}$$

where $T(x)$ is measured in millions of dollars and x is the number of months since the book's release. What will be $T(x)$ as $x \to 0$? Interpret the result.

Q 9: (a) Due to mining some oil sands, a river flowing nearby is found to be contaminated with bitumen, a dense and extremely viscous form of petroleum. Since this river is part of the water supply to a large farm region downstream from it, a proposal submitted to the farm council indicates that the cost, measured in millions of dollars, of removing $x\%$ of the toxic pollutant is given by

$$C(x) = \frac{0.5x}{100 - x} \quad (0 < x < 100)$$

What will the cost be when the percentage of pollutants to be removed approaches 100?

Q 9: (b) Suppose that a falling object reaches velocity $v(t) = 50\left(1 - e^{-\frac{t}{5}}\right)$ at time t, where distance is measured in m and time s. What is the object's terminal velocity as t goes to infinity?

Q 10: (a) The marginal cost $C'(x)$ and marginal revenue $R'(x)$ are given by

$$C'(x) = 50 + \left(\frac{x}{50}\right)$$

and $R'(x) = 60$. The fixed cost is ₹200. Find the total Profit when 500 units are produced.

Q 10: (b) Evaluate:

$$\int (2x + 5)(3x + 11)\, dx$$

Q 11: (a) Find the function $f\ (x)$ whose tangent has a slope $(3x^2 + 1)$ for each value of x and whose graph passes through the point$(2, 6)$.

Q 11: (b) Evaluate:

$$\int_0^1 (10x + 7)\ (5x^2 + 7x + 15)^3\ dx$$

Q 12: (a) A manufacturer's marginal revenue function is given by

$$MR = 275 - x - 0.3x^2.$$

Find the increase in the manufacturer's total revenue if the production is increased from 10 to 20 units.

Q 12: (b) Evaluate:

$$\int (3ln\ x + 2e^x + x^e)^{10} \left(\frac{3}{x} + 2e^x + ex^{e-1}\right)\ dx$$

****