Ahmedabad University

School of Arts and Sciences

PHY312 Quantum Mechanics-II

B.S (Physics)

Mid-Semester Examination 2022 (Winter Semester)

Total Marks = 40 **Time – 2 hours**

Enrolment Number:.....

Date: 08-02-2022

No. of printed pages: 2

Answer any 5 questions: (5x8=40)

1) Comment on whether the following is a vector space or not: [4x2=8]

(a) All $\mathbb{M}_{2\times2}$ matrices with real entries
(b) A set of all 3 degree polynomial with standard operations.

2) Find a basis for the subspace S={(a, b, c, d) ϵ $\mathbb{R}^4$ | 6a-5b-7d=0} and write down the dimension of S. [4+4=8]

3) (a) Show that the operator $|\psi\rangle\langle\psi|$ is a projection operator only when $|\psi\rangle$ is normalised. [4]

(b) Suppose $|\psi\rangle = \begin{pmatrix} 1 \\ 3i \\ 2 \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} 3i \\ -4 \\ 2i \end{pmatrix}$. Find:

(i) $\langle\psi|, |\phi\rangle^*$ [1]

(ii) Is $|\psi\rangle$ normalised? If not normalise it. [2]

(iii) Is $\langle\phi|\psi\rangle = 0$? [1]

4. Suppose you have a complete and orthonormal set of basis $\{|\phi_n\rangle\}$ and $\{|\phi'_n\rangle\}$.

(a) Write down the matrix connecting the two basis sets. [4]

(b) Suppose U is the transformation matrix connecting the two basis sets , can you comment and hence prove with proper mathematical steps and arguments whether the matrix is unitary or not? [4]

5. Consider a three dimensional vector space with orthonormal basis $|1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}, |2\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}, |3\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$. Assuming complex constants a, b and c, lets define the following kets

$$|\psi\rangle = a|1\rangle - b|2\rangle + c|3\rangle \quad \text{and} \quad |\phi\rangle = b|1\rangle + a|2\rangle$$

(a) Express $|\psi\rangle$ and $|\phi\rangle$ as column vectors in the given basis. Calculate $\langle\psi|$ and $\langle\phi|$.

(b) Show $\langle\phi|\psi\rangle = \langle\psi|\phi\rangle^*$

(c) Show the matrix representation of the operator $|\phi\rangle\langle\psi|$ in the given basis.

(d) Is the operator $|\psi\rangle\langle\psi| + |\phi\rangle\langle\phi|$ Hermitian? [8]

6. Suppose that a linear operator A, through not hermitian, commutes with its adjoint.

(a) What can be said about the relation between the eigenvalues of A and $A^\dagger$?[3]

(b) What can be said about the scalar product of two eigenstates of A with unequal eigenvalues?[5]

7. Consider a n-dimensional vector space V having two basis spaces $\{u_i\}$ and $\{v_i\}$ where $i = 1....n$. There are two linear operators: $A : u \to v$ and $B : v \to u$.

(a) Show that the matrix representation of A in $\{u\}$ basis and metric representation of B

In $\{v\}$ basis are inverses of each other. [2]

(b) Consider now an operator T in V. Find the matrix relation between $T_{ij}\{u\}$ and $T_{ij}\{v\}$ in terms the A and its inverse. [4]

(c) Show that trace of the matrix representation of T is basis independent.[2]
