Ahmedabad University

School of Arts and Sciences

PHY316 Solid State Physics

Mid Semester Examination 2025 (Winter Semester): 6 March 2025
Printed pages: 1

Total marks 25 **Time 2 hours.**

1. When a magnetic field couples to the spin of a free electron the electron energies split into two values: $\epsilon_{\mathbf{k}} \to \epsilon_{\mathbf{k}} \pm \mu B/2$, where μ is the magnetic moment of the electron, and B the applied field. The $-\mu B/2$ energy is for the up spin state (in the direction of the field) the $+\mu B/2$ energy is for the down spin state. Try the following for an electron gas in 3D:

   (a) What is the density of states (DOS) associated with up spin and down spin electrons when $B = 0$? Write the expression and plot the DOS.

   (b) For the given total density n what is the value of the Fermi energy ϵ_F when $B = 0$?

   (c) For $B \neq 0$ plot the up spin and down spin DOS. Mark the relevant scales in the plot.

   (d) Since the total number $n_\uparrow + n_\downarrow$ remains the same as at $B = 0$, what decides the Fermi energy $\epsilon_F(n, B)$? Write the equation. Can this be solved for arbitrary B?

   (e) For a given n as you increase B the magnetisation $m = n_\uparrow - n_\downarrow$ will increase. At what B does m saturate, i.e, stop increasing? Find this $B_{sat}(n)$.

   (f) Plot the DOS and locate the Fermi energy when $B = B_{sat}$. [2+1+2+2+2+1=10]

2. Consider the problem of two sites, R and R', with two levels, α and β, per site.

$$\begin{aligned} H &= \epsilon_\alpha(|R,\alpha\rangle\langle R,\alpha| + |R',\alpha\rangle\langle R',\alpha|) + \epsilon_\beta(|R,\beta\rangle\langle R,\beta| + |R',\beta\rangle\langle R',\beta|) \\ &\quad + [t_{\alpha\alpha}|R,\alpha\rangle\langle R',\alpha| + t_{\beta\beta}|R,\beta\rangle\langle R',\beta| + t_{\alpha\beta}(|R,\alpha\rangle\langle R',\beta| + |R,\beta\rangle\langle R',\alpha|) + h.c] \end{aligned}$$

   This is a model defined in the space of $|R,\alpha\rangle, |R,\beta\rangle, |R',\alpha\rangle, |R',\beta\rangle$. We assume $\epsilon_\beta > \epsilon_\alpha$ and $\epsilon_\beta - \epsilon_\alpha \gg$ the t's.

   (a) Write out the 4×4 matrix in the space of these states.

   (b) What are the eigenvalues and eigenfunctions when (i) $t_{\alpha\beta} = 0$ but other parameters are non zero, and (ii) when $t_{\alpha\alpha} = 0$, $t_{\beta\beta} = 0$, but $t_{\alpha\beta} \neq 0$.

   (c) What can you guess about the ground state if $t_{\alpha\alpha}, t_{\beta\beta} \neq 0$ & $t_{\alpha\alpha}$ is small. [4+4+2=10]

3. This is about phonons in a coupled mass spring system. If you have atoms of mass M coupled to nearest neighbours with springs of stiffness K in a one dimensional system, then lattice waves will have energy-momentum relation: $\omega_q = \sqrt{\frac{2K}{M}} sin(\frac{qa_0}{2})$ with $0 \leq qa_0 \leq \pi$.

   (a) Find the low q limit of ω_q and calculate the associated DOS in 1D.

   (b) If you are in 3D, with ω_q as in (a), and $q = \sqrt{q_x^2 + q_y^2 + q_z^2}$ find the DOS. [3+2=5]