Enrolment No. _____

AHMEDABAD UNIVERSITY

AMRUT MODY SCHOOL OF MANAGEMENT

UNDERGRADUATE PROGRAMMES

2022-2023 WINTER SEMESTER

END SEMESTER EXAMINATION

MAT142 INTRODUCTORY CALCULUS

Date: 07-05-2023 Total Marks: 60

Time: 02:00 pm – 04:30 pm No. of printed pages: 3

Instructions:

(i) Each question is compulsory. (ii) Use of a simple calculator is allowed.

(Each question carries 4 marks)

SECTION 1

[1] The XYZ Company has a fixed cost of Rs. 20,000, a production cost of Rs.12for each unit produced, and a selling price of Rs. 20 for each unit produced. Find the break-even point for the firm. If the company produces and sells 2000 units, would they obtain a profit or loss?

[2] The concentration of the drug in the patient's bloodstream t hours after an injection is $C(t)$ milligrams per millilitre where

$$C(t) = \frac{0.4t + 10}{5t + 3\sqrt{t} + 11} + 0.025$$

a. What is the concentration of the drug immediately after the injection?

b. What is the residual concentration of the drug, that is, the concentration that remains in the long run?

[3] The total cost and total revenue functions for a product are

$C(q) = 40000 + 25q + 0.002q^2$

$R(q) = 75q - 0.008q^2$

Using the marginal approach, determine the profit-maximizing level of output. What is the maximum profit?

[4] The rate of the new product is given by $f'(x) = 100 - 90e^{-x}$, where x is the number of days the product is on the market. Find the total sale during the first four days.

$(e^{-4} = 0.018)$

[5] A manufacturer of motorcycles estimates that if x thousand dollars is spent on advertising, then $M(x)$ cycles will be sold, where

$M(x) = 2300 + \frac{125}{x} + \frac{517}{x^2}, \quad 3 \le x \le 18$

At what rate will sales be changing when \$9,000 is spent on advertising? Are sales increasing or decreasing for this level of advertising expenditure?

[6] The marginal cost C'(x) and marginal revenue R'(x) are given by C'(x) = 50 + (x/50) and R'(x) = 60. The fixed cost is 200. Determine the maximum profit.

[7] After initiating an advertising campaign in an urban area, a satellite dish provider estimates that the number of new subscribers will grow at a rate given by $N'(t)$ subscribers per month where t is the number of months after the advertising begins and $N'(t) = 37 + 154t^{2/3}$. How many new subscribers should be expected 8 months from now?

[8] A management company is going to build a new apartment complex. They know that if the complex contains x apartments the maintenance costs for the building, landscaping, etc. will be,

$$C(x) = 4000 + 14x - 0.04x^2,$$

The land they have purchased can hold a complex of at most 500 apartments. How many apartments should the complex have in order to minimize the maintenance costs?

SECTION 2

[1] Evaluate the following limits:

a. $\lim_{x \to k} \frac{x^2 - \sqrt{3x^2 + k^2}}{x + k}$

b. $\lim_{x \to \infty} \frac{(12+x)^{50}(4+x)^{15}}{(2-x)^{65}}$

[2] For what values of the constants m and n, the following function is continuous at every x?

$$f(x) = \begin{cases} mx + 2n, & x \leq 0 \\ x^2 + 3m - n, & 0 < x \leq 2 \\ 5x^3 + 7x^2 - 3x - 5, & x > 2 \end{cases}$$

[3] For what values of a, $f: [0, 2] \to R$ such that

$$f(x) = 3ax \text{ when } x \geq 1,$$
$$= 9 \quad \text{when } x < 1$$

is differentiable at x = 1.

[4] Find the first-order derivative of the following function:

a. $y = \ln (x^e . e^x . 5^x . x^5 . e^e)$

b. $f(y) = \ln (1 - 5y^2 + y^3)$.

[5] Using integration, find the area of the region bounded by the line y = 1+ x, the x-axis, and the ordinates x = – 2 and x = 3.

[6] Find the indicated integral:

a. $\int \frac{(2x+3)^3}{x^2} dx$

b. $\int (20t - 6)(5t^2 - 3t + 2)^{\frac{4}{3}} dt$

[7] Find the indicated integral:

a. $\int_{-2}^{3} (2x + 3)^2 (5x - 2)\, dx$

b. $\int_{3}^{5} x^5 \ln x \, dx$

[Given: ln 5 = 1.6094 & ln 3 = 1.0986]