Ahmedabad University

School of Arts and Sciences

PHY211 - Intermediate Classical Mechanics

BS programme

Mid Term Examination 2022 (Winter Semester)

Total marks : 50 **Time : Two hours**

Enrolment Number: .....

Date: March 10, 2022 (17:00 TO 19:00)

No. of printed pages: 02

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1. Answer the following:

   (a) (1 mark) For an isolated system of particles, if the angular momentum has to be conserved, what can be say about $\vec{F}_{ij}$, $\vec{F}_{ji}$, $\vec{r}_i$, $\vec{r}_j$ etc?

   (b) (1 mark) Under what conditions are the centre of mass and centre of gravity the same?

   (c) (2 marks) The rotational energy of the earth is decreasing steadily because of tidal friction. Estimate the change in the rotational energy of the earth in a day. It is given that the rotational period of the earth decreases by about 10 microseconds in a year. Assume the earth to be a solid sphere.

2. Answer the following:

   (a) (2 marks) Consider a system consisting of three point masses: $m_1 = 1$ g, with position vector $\vec{r}_1 = (1, 5, 7)$ cm, $m_2 = 3$ g, with position vector $\vec{r}_2 = (-1, 2, 3)$ cm, $m_3 = 10$ g, with position vector $\vec{r}_3 = (0, 4, 5)$ cm. Find the coordinates of centre of mass of the system.

   (b) (2 marks) Consider a galaxy containing $N \approx 1.6 \times 10^{11}$ stars, each of mass $M \approx 2 \times 10^{30}$ kg (which is the same as mass of the Sun). We wish to estimate the gravitational potential energy of this galaxy. In order to do so, we assume that the distance between any pair of stars is $R \approx 10^{23}$ cm and ignore the gravitational self-energy of the individual stars. Given these assumptions, what will be the gravitational potential energy of the galaxy in SI units?

   (c) (3 marks) Consider an isolated system consisting of two electrons. Draw a schematic graph of the potential energy of the system as a function of separation between them. Do the same for effective potential energy. Can we say anything about the shape of the orbit?

   (d) (2 marks) Show that one possible parametric form of the equation of an ellipse is $x = a\cos\theta$ and $y = b\sin\theta$.

   (e) (3 marks) The polar equation of a conic section is sometimes written as $r = \frac{s}{1+e\cos\theta}$ and at other times written as $r = \frac{s}{1-e\cos\theta}$. Under what conditions is each of them applicable?

3. Consider the motion of a two-particle system under the influence of a mutual central force. Let the reduced mass of the system be ℓ and let the force law be $f(r)$.

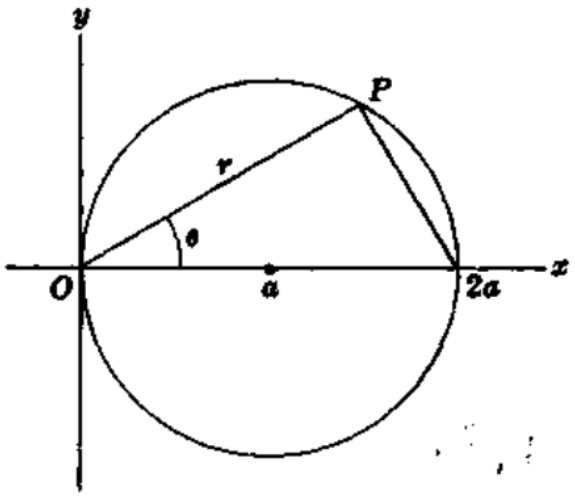


Figure 1: The trajectory in problem 3.

(a) (5 marks) Show that, for $u = 1/r$,

$$\frac{d^2u}{d\theta^2} + u = \frac{-\mu}{\ell^2 u^2} f\left(\frac{1}{u}\right) , \tag{1}$$

(b) Suppose that one of the particles is too heavy as compared to the other. Under the influence of a central force at point O, where the heavier particle is located, the lighter particle moves in a circular orbit which passes through O, see Fig (1).

i. (2 marks) Write the polar equation of the circle shown in Fig (1).

ii. (5 marks) Find the law of force.

4. A rigid body consists of three particles of masses 2, 1, 4 located at $(1, -1, 1)$, $(2, 0, 2)$ and $(-1, 1, 0)$.

(a) (3 marks) Find the angular momentum of the body if it is rotated about the origin with angular velocity $\vec{\omega} = (3, -2, 4)$.

(b) (6 marks) Determine the moments of inertia about x, y and z axis and the products of inertia.

(c) (3 marks) Calculate the rotational kinetic energy of the system.

5. (10 marks) A thick hollow sphere, with inner radius R_1 and outer radius R_2, rolls without slipping on an inclined plane which makes an angle θ from the horizontal. Find an expression for its linear acceleration. Justify and clarify each step of your calculation.