General Relativity, Black-holes & Cosmology
PHY410
17^{th} September, 2022.
Time: 5:00pm-7:00pm

Midsem

All questions are compulsory. Each question carries 10 marks.

Question 1.

1) Define an Inertial Frame of Reference. Give two examples of frames of references which are not inertial frames.
2) Explain how classical momentum changes in special relativity. Derive the expression for relativistic momentum.

Question 2. Prove that the given equations are Lorentz invariant:

$$\text{a) } \nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0, \qquad \text{b) } \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Question 3.

1) Calculate the speed v at which the kinetic energy of a particle becomes $\frac{1}{2}$ of its rest mass energy.
2) Muons are unstable particles created when cosmic rays interact with the upper atmosphere. They move at very high velocities ($\beta \sim 0.9999$) and have very short lifetimes, $\tau = 2\times10^{-6}s$, as measured in the lab. Still muons manage to reach the ground, given an atmospheric "thickness" of about $10km$. How is it possible? Try to explain in two different ways.

Question 4.

1) Prove that a 2-sphere is a differentiable manifold.
2) Give examples of two chart systems or atlases for the same.

Question 5.

1) Consider a light beam passing through a horizontal column of water moving with velocity v.
   (a) Determine the speed u of the light measured in the lab frame when the beam travels in the same direction as the flow of the water.
   (b) Determine an approximation to this expression valid when v is small.
2) Explain time-dilation and length-contraction using space-time diagram.