Enrolment No. _____

AHMEDABAD UNIVERSITY
AMRUT MODY SCHOOL OF MANAGEMENT
GRADUATE PROGRAMMES
2025-2026 MONSOON SEMESTER
MID SEMESTER EXAMINATION
TOD 504 MATHEMATICAL METHODS FOR ECONOMICS

Date: 21-09-2025
Time: 8.00 am – 9.30 am

Total Marks: 50
No. of printed pages: 02

Instructions
- You can carry a cheatsheet of A4 page containing handwritten notes of the theorem, lemma or results.
- Please attempt exactly 10 problems.
- Justify your arguments and mention all the relevant theorem or results used in your solution.
- Each question carries 5 points.
- All the best!

Problem 1: For any positive integer n, prove that the binomial coefficient $\binom{2n}{n}$ is an even integer.

Problem 2: Find a closed expression for the following sum

$$\sum_{i=1}^{n} \binom{n}{i} (i^3 + i^2 + 1).$$

Problem 3: In how many ways can you take an odd number of objects from n objects.

Problem 4: If $x > 0$, then show the inequality

$$1 + \frac{1}{2}x - \frac{1}{8}x^2 \leq \sqrt{1+x} \leq 1 + \frac{1}{2}x.$$

Or

Use Taylor's theorem to calculate $\sqrt[4]{1.3}$ with error less than 10^{-5}.

Problem 5 (2.5+2.5): Evaluate the following limits

$$\lim_{x \to 0+} \frac{ln(1+x)}{sin(x)} \text{ and } \lim_{x \to 2} \frac{x^4 - 4^x}{sin(\pi x)}.$$

Problem 6: Find the mistake in the following proof of Cauchy's mean value theorem

Let f and g are continuous on [a, b] and differentiable on (a, b) and assume that $g'(x) \neq 0$ for all $x \in (a, b)$. Then there exists a number c such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}.$$

Step 1: Since the conditions are met for the "mean value theorem" on f and g, we can conclude that there exists c such that $\frac{f(b)-f(a)}{b-a} = f'(c)$ and $\frac{g(b)-g(a)}{b-a} = g'(c)$.

Step 2: Since the derivative of g is given to be non-zero everywhere, dividing the given equations we can easily conclude that there exists c such that $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$.

Problem 7: Differentiate the following functions by implicit differentiation
$(x-y)^3 + y^2 = x + y - 1$ and $y^2 + x^2 = sin(2x+3y)$.

Problem 8 (1+1.5+2.5): Let $I = [0,\pi/2]$. Define a function $f(x) = max\{x^2, cosx\}$. Show that f is continuous. Show that there exists an absolute minimum point x_0 for f and $x_0 \in I$. Show that the absolute minimum x_0 satisfies the equation $cos(x) = x^2$.

Problem 9: f and g are two continuous functions on $\mathbb{R}$ such that $f(r) = g(r)$ for all rational numbers r. Is is true that they are equal elsewhere? Is yes provide a proof. If not provide a counter-example.

Problem 10: Define the function f given by the following rules

$$f(x) = \begin{cases} |x-1| \text{ if } x \leq 0 \\ x^2 + 1 \text{ if } 0 < x \leq 2 \\ |x-5| + |2-x| \text{ if } x > 2 \end{cases}$$

Determine all the points (with justification) where the function is discontinuous.

Problem 11: Show that the limit $\lim_{x\to 0}\left(sin(\frac{1}{x}) + x\right)$ does not exist.

Problem 12: Evaluate the following series $\sum_{n=0}^{\infty} \frac{1}{(2+n)(3+n)}$.

Problem 13: Aayush invested 3000 in a bank that pays an annual interest rate of 7% compounded continuously. What is the amount she can get after 5 years from the bank? Round your answer to the nearest integer.