PHY701 Mathematical Methods for Physics
Mid-Semester Examination
Monsoon Semester 2022
Professor Raghavan Rangarajan

Total Marks = 40 Time – 2 hours
Enrolment Number: .....
Date: 18-09-2022
No. of printed pages: 1

Each problem carries equal weightage.
Start each problem on a new page.
Your solutions should be clear.
Include intermediate steps.

1. Obtain the leading order behaviour as $x \to \infty$ of

$$I(x) = \int_0^\infty \ln(1+t) e^{-x \sinh^2 t}\, dt$$

2. Obtain the leading order behaviour as $x \to 0$ of the Stieltjes integral

$$I(x) = \int_0^\infty \frac{1}{1+xt} e^{-t}\, dt$$

3. For integer n, the Bessel function $J_n(x)$ has the integral representation

$$J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(x \sin t - nt)\, dt$$

   Obtain the leading order behaviour of $J_n(n)$ as $n \to \infty$.

4. Discuss the analytic properties of the function $f(z) = \tan(1/z)$ at $z = 0$. You may assume f is analytic outside of the real axis.

Possibly useful formulae:

$$\int_0^\infty e^{-a^2 x^2}\, dx = \sqrt{\pi}/(2a)$$

$$\int_0^\infty e^{\pm i x^p}\, dx = (1/p)\Gamma(1/p)\, e^{\pm i\pi/(2p)}$$

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx \qquad Re(z) > 0$$

If $I_n = \int_0^\infty t^n \mathrm{e}^{-xt^2}\, dt$, $I_{2m} = \frac{1}{2}\sqrt{\pi}(-1)^m \frac{d^m}{dx^m}(x^{-\frac{1}{2}})$ and $I_{2m+1} = \frac{1}{2}(-1)^m \frac{d^m}{dx^m}(x^{-1})$