Ahmedabad University

School of Arts and Sciences

PHY310 - Quantum Mechanics - I

BS programme

Mid Term Examination 2022 (Monsoon Semester)

Total marks : 35 **Time : Two hours**

Enrolment Number: .....

Date: September 20, 2022 (05:00 PM TO 07:00 PM)

No. of printed pages: 02

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1. (3+3 = 6 marks) Show that

   (a)
   $$\int_{-\infty}^{+\infty} dx\ e^{-x^2} = \sqrt{\pi}\ .$$

   (b)
   $$\int_{-\infty}^{+\infty} dx\ e^{-ax^2-ibx} = \sqrt{\frac{\pi}{a}}\ e^{\frac{-b^2}{4a}}\ .$$

2. Consider a free particle in one dimensional QM with initial wave function

   $$\Psi(x,0) = A\ e^{\frac{-(x-\mu)^2}{2\sigma^2}}\ ,$$

   (a) (1 mark) Normalise $\Psi(x,0)$.

   (b) (1 mark) Write down the expression for $|\Psi(x,0)|^2$.

   (c) (1 mark) What is the maximum value of the $|\Psi(x,0)|^2$?

   (d) (1 mark) At what value(s) of x is the value of $|\Psi(x,0)|^2$ maximum?

   (e) (1 mark) At what value(s) of x is the value of $|\Psi(x,0)|^2$ equal to e^{-1} of its maximum value?

   (f) (5 marks) Find $\Psi(x,t)$.

3. A particle in the infinite square well has as its initial wave function an even mixture of the first two stationary states

   $$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]\ ,$$

   (a) (2 marks) Normalize $\Psi(x,0)$.

   (b) (3 marks) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter as a sinusoidal function of time

   (c) (5 marks) Compute $\langle x\rangle$. Notice that it oscillates in time. What is the angular frequency of the oscillation? What is the amplitude of the oscillation?

   (d) (1 mark) Compute $\langle p\rangle$.

(e) (1 mark) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation value of H. How does it compare with E_1 and E_2 ?

4. Consider the Hamiltonian of an electron in an atom

$$H = \frac{p^2}{2m_e} - \frac{Zq^2}{4\pi\epsilon_0 r} \ .$$

Suppose the wavefunction of the electron is given by $\Psi(\vec{x}, L)$ where, L is the length scale over which Ψ varies significantly.

(a) (2 marks) Argue that the expectation value of the Hamiltonian in this state Ψ is

$$E(L) = \frac{\hbar^2}{2m_e L^2} - \frac{Zq^2}{4\pi\epsilon_0 L} \ .$$

(b) (3 marks) What is the expression for the value of L for which this expression for $E(L)$ takes up its minimum possible value? Find this value in cm.

(c) (2 marks) What is expression for the corresponding minimum value of energy? Find this value in eV.