Enrolment No. _____

AHMEDABAD UNIVERSITY
AMRUT MODY SCHOOL OF MANAGEMENT
UNDERGRADUATE PROGRAMMES
2023-2024 WINTER SEMESTER
END-SEMESTER EXAMINATION
STA100 PROBABILITY

Date: 26-04-2024
Time: 2:00 pm to 4:00 pm

Total Marks: 50
No. of printed pages: 12

Question 1: Do as directed: (20 Marks)

[A] Dataset:

Apartment No.	yr_built	price	bedrooms	bathrooms	sqft_living	floors
1	1955	231300	2	1	1180	1
2	1951	538000	3	2	2570	2
3	1933	180000	2	1	770	1
4	1965	604000	4	3	1960	1
5	1987	510000	3	2	1680	1
6	2001	1230000	4	5	5420	1
7	1995	257500	3	2	1715	2
8	1963	291850	3	2	1060	1
9	1960	229500	3	1	1780	1
10	2003	323000	3	3	1890	2
11	1965	662500	3	3	3560	1
12	1942	468000	2	1	1160	1
13	1927	310000	3	1	1430	2
14	1977	400000	3	2	1370	1
15	1900	530000	5	2	1810	2
16	1979	650000	4	3	2950	2
17	1994	395000	3	2	1890	2
18	1916	485000	4	1	1600	2
19	1921	189000	2	1	1200	1
20	1969	230000	3	1	1250	1
21	1947	385000	4	2	1620	1
22	1968	2000000	3	3	3050	3
23	1995	285000	5	3	2270	2
24	1985	252700	2	2	1070	1
25	1981	289000	4	3	1500	3

Answer the following questions based on the given dataset.

[1] A random variable X denotes the number of bedrooms. Determine the probability distribution of X, and answer the following questions: (10 Marks)

(a) Compute the expected number of bedrooms.

(b) Determine the variance of X.

(c) Evaluate: $E(2X + 3)$, $E(3X^2 + 5X + 2)$, $V(2X + 3)$, $V(3 - 2X)$.

[2] Eight apartments are selected at random at a time. Determine the probability that: (5 Marks)
(i) at least six apartments have two bedrooms.
(ii) at most two apartments have three bedrooms.

[3] Twenty apartments are selected at random at a time. Determine the probability that: (5 Marks)
(i) Exactly two apartments have five bathrooms.
(ii) at most three apartments have five bathrooms.

Question 2: Do as directed: **(30 Marks)**

[1] As a result of tests on 20,000 electric bulbs manufactured by a company, it was found that the lifetime of the bulb was normally distributed with an average life of 2,040 hours and a standard deviation of 60 hours. On the basis of the information, estimate the number of bulbs that are expected to burn for (a) More than 2,150 hours (b) between 2,175 hours and 2220 hours.

(5 Marks)

[2] Suppose that in an electric display sign, there are two light bulbs in the first row and three light bulbs in the second row. Let X denote the number of bulbs in the first row that will be burned out at a specified time t, and let Y denote the number of bulbs in the second row that will be burned out at the same time t. Suppose that the joint probability mass functionm of X and Y is as specified in the following table:

X	Y			
	0	1	2	3
0	0.08	0.07	0.06	0.09
1	0.11	0.08	0.07	0.05
2	0.09	0.12	0.12	0.06

Compute Cov(X, Y) and ρ(X, Y). Interpret your results: (5 Marks)

[3] An apartment manager decides to see if timely payment of rent is related to unpaid credit card balances. Using X as the number of unpaid credit card balances, and Y as the number of timely rent payments in the last four months, he examines his records and the credit reports of his 100 renters and obtains the following results. (5 Marks)

	X=0	X=1	X=2	X=3
Y=0	0.01	0.01	0.06	0.03
Y=1	0.02	0.04	0.07	0.04
Y=2	0.03	0.08	0.09	0.04
Y=3	0.05	0.11	0.08	0.02
Y=4	0.13	0.05	0.03	0.01

Find the followings:

a) What is the number of renters with exactly 3 timely payments? (1 mark)

b) What is the number of renters with exactly 1 unpaid credit card balance? (1 mark)

c) Compute the expected number of unpaid credit card balances known that no timely rent payments in the last four months. (3 marks)

[4] The data below shows the outcomes of vehicle crashes when using different types of safety equipment. The random variable X corresponds to the level of injury and the random variable Y corresponds to the type of safety equipment used. (5 Marks)

	X=None (0)	X= Minor (1)	X=Major (2)	X= Death (3)
Y= None (0)	0.0165	0.175	0.135	0.025
Y= Belt Only (1)	0.075	0.16	0.1	0.015
Y= Belt + Airbag (2)	0.06	0.115	0.065	0.01

Use the above information to answer the following questions:

a) If you sustained minor injuries, what is the probability that you had a seat belt and an airbag?

b) If you used a seat belt and an airbag, what is the probability that you sustained minor injuries?

[5] Suppose we have a population with heights of adult males (in feet) following a probability density function given as follows: (5 Marks)

$$f(x) = \begin{cases} \frac{1}{45}\left(2x + \frac{3}{2}\right) & 0 \leq x \leq 6 \\ 0 & Otherwise \end{cases}$$

Find the probability that

a) A randomly selected adult male from this population has a height between 4 to 6 feet.
b) A randomly selected adult male from this population has a height less than 3 feet.

[6] The p.d.f. of the cholesterol ratio of LDL and HDL of Mr. X, being continuously noted by a doctor of Sanjivani Hospital, is given by (5 Marks)

$$f(x) = \begin{cases} C\ (x^2 + 5x - 7) & 2 < x < 4 \\ 0 & Otherwise \end{cases}$$

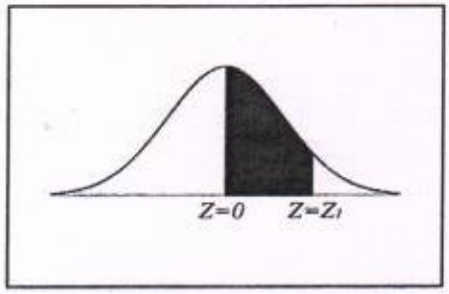
a) Compute the value of C.

b) What is the probability that his cholesterol ratio will remain between 1 to 3?

Table for e^{-x} values

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0	1.0000	3.0	0.0498	6.0	0.00248	9.0	0.00012
0.1	0.9048	3.1	0.0450	6.1	0.00224	9.1	0.00011
0.2	0.8187	3.2	0.0408	6.2	0.00203	9.2	0.00010
0.3	0.7408	3.3	0.0369	6.3	0.00184	9.3	0.00009
0.4	0.6703	3.4	0.0334	6.4	0.00166	9.4	0.00008
0.5	0.6065	3.5	0.0302	6.5	0.00150	9.5	0.00007
0.6	0.5488	3.6	0.0273	6.6	0.00136	9.6	0.00007
0.7	0.4966	3.7	0.0247	6.7	0.00123	9.7	0.00006
0.8	0.4493	3.8	0.0224	6.8	0.00111	9.8	0.00006
0.9	0.4066	3.9	0.0202	6.9	0.00101	9.9	0.00005
1.0	0.3679	4.0	0.0183	7.0	0.00091	10.0	0.00005
1.1	0.3329	4.1	0.0166	7.1	0.00083		
1.2	0.3012	4.2	0.0150	7.2	0.00075		
1.3	0.2725	4.3	0.0136	7.3	0.00068		
1.4	0.2466	4.4	0.0123	7.4	0.00061		
1.5	0.2231	4.5	0.0111	7.5	0.00055		
1.6	0.2019	4.6	0.0101	7.6	0.00050		
1.7	0.1827	4.7	0.0091	7.7	0.00045		
1.8	0.1653	4.8	0.0082	7.8	0.00041		
1.9	0.1496	4.9	0.0074	7.9	0.00037		
2.0	0.1353	5.0	0.0067	8.0	0.00034		
2.1	0.1225	5.1	0.0061	8.1	0.00030		
2.2	0.1108	5.2	0.0055	8.2	0.00027		
2.3	0.1003	5.3	0.0050	8.3	0.00025		
2.4	0.0907	5.4	0.0045	8.4	0.00022		
2.5	0.0821	5.5	0.0041	8.5	0.00020		
2.6	0.0743	5.6	0.0037	8.6	0.00018		
2.7	0.0672	5.7	0.0033	8.7	0.00017		
2.8	0.0608	5.8	0.0030	8.8	0.00015		
2.9	0.0550	5.9	0.0027	8.9	0.00014		

Table of Standard Normal Curve



Area Under the Standard Normal Curve

Z = 0 to Z = Z_1 , z being standard normal variate

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4762	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998