Enrolment No. _____

AHMEDABAD UNIVERSITY

AMRUT MODY SCHOOL OF MANAGEMENT

GRADUATE PROGRAMME

2023-2024 WINTER SEMESTER

END SEMESTER EXAMINATION

FAC 639 MODELLING RANDOMNESS IN FINANCIAL MARKETS

Date: 21-04-2024
Time: 5.00 p.m. – 7.30 p.m.

Total Marks: 50
No. of printed pages: 03

Question 1 (Answer in not more than two sentences)
How would the Black-Scholes model be characterised in terms of

a) classification (discrete vs. continuous and deterministic vs. stochastic)
b) type of differential equation (ordinary / stochastic / partial)

(2 marks)

Question 2 (Answer in not more than two sentences)
Consider statements A) and B) given below.

A) Whenever markets are open, there exists some volatility.
B) Uncertainty of asset prices does not vanish when markets are observed during small time intervals.

Which of the above two statements is 'True' and which of them is 'False'?

(2 marks)

Question 3 (Answer in not more than two sentences)
What is a continuous square integrable martingale?

(2 marks)

Question 4 (Answer in not more than two sentences)

a) Which moments of a Wiener process are relevant?
b) Which moments of rare events are relevant?

(2 marks)

Question 5 (Answer in not more than four sentences)
Consider a Wiener process W_t.

a) What is the mean of W_t?
b) Is W_t a martingale?
c) What is the variance of W_t?
d) Does W_t exhibit any jumps?

(4 marks)

Question 6 (Answer in not more than four sentences)

a) Give an example of a random variable that is i.i.d. (independent, identically distributed). Explain these two terms.
b) What can be considered as the limit of independent, identically distributed random variables?

(4 marks)

Question 7 (Answer in not more than four sentences)

a) What are the two characteristics on the basis of which one can differentiate a normal event from a rare event?

b) In the case of a normal event, what happens to these two characteristics when the time interval decreases?

(4 marks)

Question 8 (Answer in not more than four sentences)

Interpret $E_{k-1}[S_k - S_{k-1}] = A(I_{k-1}, h)$

(3 marks)

Question 9 (Answer in not more than four sentences)

While deriving the Black-Scholes equation, a risk-free portfolio is formed in continuous time.

a) What is the expression (equation) for the value of this portfolio? Explain the terms.

b) What is the expression (equation) for the change in portfolio value? Explain the terms.

(4 marks)

Question 10 (Answer in not more than four sentences)

Distinguish SDE from PDE by taking an example for each.

(3 marks)

Question 11 (Answer in not more than four sentences)

State why turbulence (unsteady movement) in behaviour of an asset price cannot be considered as a rare event.

(2 marks)

Question 12 (Answer in not more than six sentences)

You are tracking the stock price of a blue-chip company and are interested in finding out the strong solution of the stochastic differential equation (SDE) that represents the stock price.

a) What would be known to you (given) to enable you arrive at the solution?

b) What would you determine?

If you were asked to determine the weak solution to the SDE, would there be any changes to a) and b) above?

(3 marks)

Question 13 (Answer in not more than six sentences)

While modeling asset price dynamics in discrete time, the binomial model is utilized. Both 'up' and 'down' movements are modeled using factors relating to drift (μ), volatility (σ) and the time interval (Δ)

a) What is the expression for the upward movement 'u'"

b) What is the expression for the downward movement 'd'?

c) What is the expression for the probability 'p' of the upward movement?

(3 marks)

Question 14

Let W_t be a random term and 't' be a deterministic term.

Consider the function $f(W_t, t) = 3W_t^2 + 2t^3$.

Apply Ito's Lemma to $f(W_t, t)$.

(5 marks)

Question 15
A particular stock's movement is as follows:

- if it goes up the previous day, there is 25% chance for it to go up the next day and 75% chance for it to go down the next day.
- if it were to remain constant the previous day, then it will move up with 100% probability.
- if it were to go down the previous day, there is 50% chance for it to go up the next day, 25% chance for it to remain constant the next day and 25% chance for it to go down the next day.

Assume that 1 represents "UP" state, 2 represent "CONSTANT" state and 3 represents "DOWN" state.

Part a)
What is the transition matrix of the Markov chain?

Part b)
Suppose the initial probability distribution is $p^{(0)} = (½ , ½ , 0)$
Explain what the following mean and compute the value of each
(i) $p_{13}^{(2)}$
(ii) $p_{23}^{(2)}$
(iii) $p^{(2)}$
(iv) $p_1^{(2)}$

(4 marks)

Question 16 (Answer in not more than six sentences)
How would you distinguish the 'Geometric SDE' from the 'Square Root Process SDE'? While answering the question, be specific with respect to the following

a) the functional form (the mathematical depiction)
b) the trend exhibited by the drift
c) the variation around the trend

(3 marks)