Differential Equations (MAT256)

Mid-semester Examinations **(Set 1)** 19th September, 2022
F.M.: 50 Time: 1 hour 55 minutes

1. Solve the differential equation $\frac{dy}{dx} = \frac{xy+3x-y-3}{xy-2x+4y-8}$. **(9)**

2. Show that $y = e^{x^2} \int_0^x dt \ e^{t-t^2}$ is a solution of the differential equation $\frac{dy}{dx} - 2xy = e^x$. **(You need to show the details of the differentiation; stating the final result/or reference to theorems without proof will not be acceptable.)** **(7)**

3. Solve the equation $x\frac{dy}{dx} + (3x+1)y = e^{-3x}$ and give the largest interval I over which the general solution is defined. **(Using any formula will not be acceptable. You must derive all steps.)** **(9)**

4. (a) Draw the direction field for $\frac{dy}{dx} = 1 - xy$. (**Mark special lines, if any, on the diagram.)**

   (b) On the direction field, draw the solution curve that passes through the point $y(0) = 0$. **(The reason for drawing a particular solution curve should be evident from your diagram.)** **(15)**

5. (a) Find the critical points and phase portrait of the differential equation $\frac{dy}{dx} = y^2 - 3y$. **(10)**

   (b) Classify each critical point as stable, unstable and semi-stable.

   (c) Sketch typical solution curves in the regions in the xy-plane determined by the graphs of the equilibrium solutions.