Ahmedabad University

School of Arts and Sciences

MAT 384 Introduction to Differential Geometry

All Programmes

Mid Term Examination 2022 (Winter Semester)

Total Marks = 100 **Time – 1.5 hours**

Enrolment Number:.....
Date: 11-03-2022
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1. A) For the parametric curve $\vec{x}(t) = (-t + 5, t^2 - 1)$, with $-1 \leq t \leq 1$ find the velocity, acceleration, arclength and curvature function $\kappa_g(t)$
(HINT: Recall that

$$\kappa_g(t) = \frac{x'(t)y''(t) - x''(t)y'(t)}{(x'(t)^2 + y'(t)^2)^{3/2}}.$$

)

B) Is $\vec{x}(t)$ a regular plane curve? If not, where are the critical points of $\vec{x}(t)$.

C) Find the unit tangent vector and unit normal to this curve. Write down the Tangential Indicatrix of

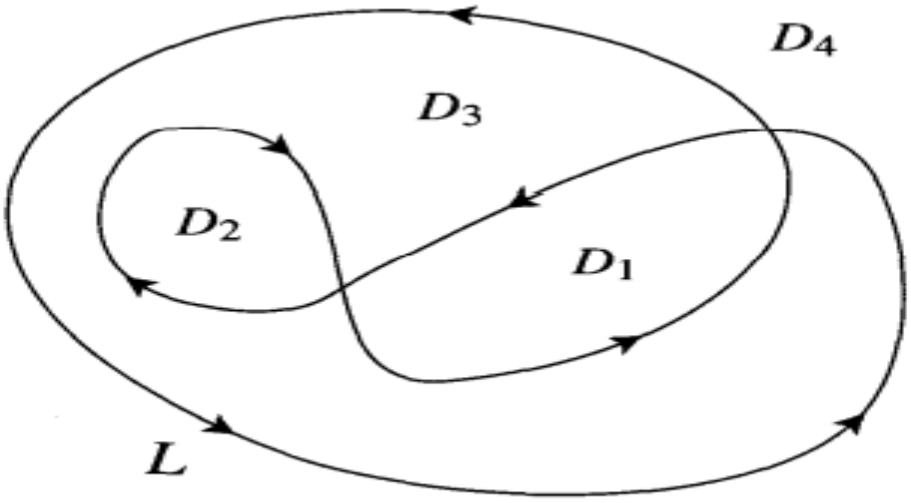
Curve as a function of t.

D) Write down the Serret-Frenet equations for this curve i.e. write down the equations obeyed

By $T'(t), U'(t), T(t), U(t)$ possibly with other factors present.

2. Suppose I was trying to draw a plane curve on a CAD/CAM software. I found that the total curvature of
   the curve was $\sqrt{2}\,\pi$. Can I draw such a curve? If not, explain why not?

3. A) Calculate the winding numbers of the curve L in the figure below around the points D_1, D_2, D_3, D_4- Explain your answers.



   B) Calculate the rotation index of the curve L above. Does it depend on points which are not on the curve L?

4. Consider a line in three-dimensional space passing through the point $\vec{p} = (1,1,-1)$ in the direction $\vec{v} = (1,2,1)$

A) Write down the parametric equations for this line
B)Calculate the equation of the Tangential Indicatrix for this line

5. A) Suppose I was trying to define a knotted, closed space curve by a parametric equation $\vec{x}(t)$ for a computer
Graphics project. According to my calculations the total curvature of the curve is 2π. Can I give an assurance that the
Curve is knotted? Why or Why not?

B)Consider the following pair of linked space curves:



Can you calculate the linking number of the above link?

6. Consider the following space curve

$$\vec{x}(t) = (2t + 10, t^2 - 1, t^3 + 5)$$

A) Find the velocity, speed and acceleration as functions of t for this space curve

B) Determine the Unit Tangent Vector, the Unit Normal Vector and the Binormal vector as a function of t

C) Calculate the arclength of this curve

D) Calculate the curvature of this curve

E) Calculate the torsion of this curve

F) Write down the Serret-Frenet Equations for this curve

(HINT: The curvature of a space curve in three-space is

$$\kappa(t) = \frac{\|\vec{x}'(t) \times \vec{x}''(t)\|}{\|\vec{x}'(t)\|^3}.$$

And its torsion is

$$\tau(t) = \frac{(\vec{x}'(t) \times \vec{x}''(t)) \cdot \vec{x}'''(t)}{\|\vec{x}'(t) \times \vec{x}''(t)\|^2}.$$

)

7. Calculate the equation of the tangent plane to the torus

$$\vec{X}(u, v) = ((2 + \cos(v))\cos(u), (2 + \cos(v))\sin(u), \sin(v))$$

At the point $(\frac{\pi}{6}, \frac{\pi}{3})$.