Ahmedabad University

School of Arts and Sciences

MAT485/585 Introduction to Quantum Computing

Mid Term Examination 2021-22 (Winter Semester)

Total Marks = 100 **Time - 2 hours**

Name & Enrolment Number:
Date:
No. of printed pages: 3

1. (10 points) Implement the Swap Gate, which takes the input state $|a, b\rangle$ to the output state $|b, a\rangle$, using three C-NOT gates.

2. (15 points)

   a. Explain the working of the Toffoli gate. Write down its transformation matrix using the standard computational basis.

   b. Explain the working of C-NOT gate. Write down its transformation matrix using the standard computational basis.

   c. It is known that NAND gates are universal gates for deterministic classical computing. Show that the Toffoli gate can be used to simulate the NAND gate.

3. (10 points) In order to simulate a random fair coin toss for a cricket match between India and Australia you need to design a quantum circuit. Assume that a qubit in the initial state $|0\rangle$ is given, and you are allowed to use any standard quantum gate.

4. (15 points) The state vector describing a two qubits system is

$$|\Psi\rangle = \frac{i}{3}\left[|00\rangle + |01\rangle + |10\rangle\right] + \sqrt{\frac{2}{3}}\,|11\rangle\,.$$

It is known that $M_{00} = |00\rangle\langle 00|$, $M_{01} = |01\rangle\langle 01|$, $M_{10} = |10\rangle\langle 10|$ and $M_{11} = |11\rangle\langle 11|$ form a set of measurement operators.

a. Write down the transformation matrix for each of the M_{00}, M_{01}, M_{10}, and M_{11}.

b. Prove the completeness relation, i.e., $M_{00} + M_{01} + M_{10} + M_{11} = I_4$. Here I_4 is the 4×4 identity matrix.

c. If the first qubit is measured and found to be 0, then what would be the post-measurement state? Use an appropriate measurement operator to obtain the post-measurement state.

5. (20 points) Suppose a two qubits system is initially in the state

$$|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle,$$

with $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Compute the following. Assume H to be the Hadamard gate.

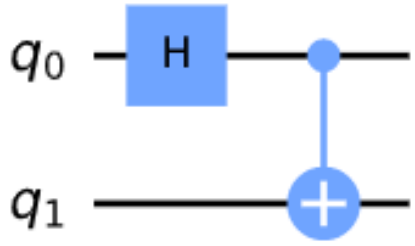
a. $\langle\Psi|\Psi\rangle$.

b. $|\Psi\rangle \otimes |\Psi\rangle$.

c. $(H \otimes H)|\Psi\rangle$.

d. The transformation matrix of $|\Psi\rangle\langle\Psi|$.

6. (10 points) (a) If the input to the circuit below is $|00\rangle$, i.e., $q_0 = |0\rangle$ and $q_1 = |0\rangle$ then



find the output state, say $|\psi\rangle$.

(b) Show that the output state is an entangled state, i.e., $|\psi\rangle \neq |\psi_0\rangle|\psi_1\rangle$ for any single qubit states $|\psi_0\rangle$ and $|\psi_1\rangle$.

7. (10 points) State and prove the "no cloning" theorem.

8. (10 points) Compute $H^{\otimes n}(|0\rangle^{\otimes n})$ for $n = 3$ and $n = 4$.

9. (10 points) [Bonus Problem] Suppose a particle of mass m is trapped in an infinite potential well of length L. The potential energy is given as

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq L \\ \infty & \text{otherwise.} \end{cases}$$

a. Show that the energy of the particle must be quantized. Find the energy E_n in the nth stationary state (the ground state is E_1).

b. Compute the probability of finding the particle in the region from $0 \leq x \leq \frac{L}{2}$.

End of Exam.
