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Lift force on single bubbles in subcooled pool boiling systems: Experimental measurements and CFD simulations

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Abstract. Lift coefficient has been evaluated for a subcooled pool boiling system with water as continuous phase and vapor (steam) bubble as dispersed phase generated on vertical walls with constant heat flux for a specific Rayleigh number (Ra) range ($8x10^{12}$< Ra < $2x10^{13}$) for the first time. Experimental investigation has been done with High Speed Camera while two phase CFD simulations have been performed using Volume of Fluid (VOF) method. The Morton number (M) for this system is $\log_{10}M$ = -10.6 and Eötvös number (Eo) range is 0.2 < Eo < 1.3 while the shear rates have been maintained in the range 2 ≤ Sr ≤ 2.8. Bubble size of the bubbles arising from the sites on vertical surface have been determined by both experiments and simulations. The CFD predictions for bubble size and lift coefficients agree well with experimental measurements with a deviation of less than 10%. Further, empirical expressions from experimental data for the estimation of bubble size and lift coefficients have also been presented. The data for lift coefficients obtained is different from the ones of bubbles generated by single sparger in air-water systems.

1. Introduction

Boiling phenomena has been studied predominantly on horizontal plates or tubes in saturated stream both using experimental and numerical techniques. Studies of boiling on vertical surfaces have been limited in the reported literature. Modeling boiling using CFD is carried out using two approaches nmely Euler-Euler and Euler-Lagrange approaches. In the Euler-Euler approache the interactions between vapor and liquid phase is accounted by a source term representing an overall volumetric force FGL in the momentum equation. This force is the summation of all interfacial forces namely drag, lift, turbulent dispersion force and wall lubrication force. The vapor bubble formation has the following steps: (i) it forms a nucleation site on the vertical surface (ii) grows to a certain size (iii) departs from the wall after attaining a certain size.

The overall volumetric force F_{GL} is given by the equation below,:

$$F_{GL} = F_D + F_L + F_{TDP} + F_{WLF} \quad (1)$$

where drag (F_D), lift (F_L), turbulent dispersion force F_{TDP} and wall lubrication force (F_{WLF})

Researchers have observed that as the bubble departs from the nucleation site (in case of orifices placed at the center at bottom of a liquid column), the bubble may take a central path parallel to the

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wall at the center or deviating away or nearer to the wall depending on the shear induced on the bulk liquid. The side way movement during its rise is due to the fact that during its motion the bubble experiences a lift force. The lift force investigations in the presence of thermal gradients however differ from the shear induced flows. Both systems however have velocity gradients in the continuous phaseand lift force plays a crucial role in determining the lateral migration of bubbles.

1.1. Previous work

Estimation of lift force in terms of experimental measurements have been done in the gas-liquid flows while works with vapor-liquid flows have been limited. The lift force exerted on a single bubble rising in a liquid is generated by three different mechanisms: magnus lift due to bubble rotation [1-3], Saffman lift force on non-rotating bodies due to shear in the liquid [4-6] and wake induced lift [7] caused by bubble wakes [8]. The most important parameters in the lift force evaluation are lift coefficient, bubble diameter and the velocities of both phases. The details of gas/vapor liquid (GL) systems on which experimental measurements and analytical modelling have been carried out, the experimental operating conditions and geometric details, typical range of bubble sizes in the GL systems and the range of lift coefficients estimated has been tabulated in Table 1 while the prominent correlations for lift coefficients have been listed in Table 2. The shear induced lift force is defined as [9-11]:

$$F_L = -C_L \rho_L \frac{\pi d_B^{\,2}}{6}(V_V - V_L)\times(\nabla \times V_L) \tag{2}$$

where F_L is the lift force, C_L is the lift coefficient, ϱ_L is the liquid density, V is the velocity and d_B is the bubble diameter.

Auton [10] carried out theoretical investigations to arrive at an expression for C_L.He assumed a clean spherical bubble in an inviscid rotational system. However, predicted value of lift coefficient (C_L=0.5) by Auton [10], did not match with the experimental observations of Kariasaki[7]. The variation in the value of C_L indicated that the lift coefficient was strongly dependent on the flow condition. Later, Tomiyama et al. [6] performed experiments to estimate the lift force acting on a single spherical/ellipsoidal air bubble of different sizes ($2.8 \le d_B \le 5.68$ mm) rising in a simple shear flow $\left(0 \le \omega \le 6.2 s^{-1}\right)$ in a viscous liquid solution (μ_L = 0.018 - 0.089 kg/m.s). The authors proposed different correlations to calculate C_L, which were valid only for viscous systems. The applicability for less viscous liquids like water was however not verified. Bothe et al. [12] performed VOF simulations of air bubble rising in viscous liquid imposed with linear shear corresponding to the experiments reported by Tomiyama et al. [6]. The predicted C_L for viscous system was in good agreement with the measured C_L of Tomiyama et al. [6]. The authors also investigated the effect of bubble diameters (range of 2.8 - 10 mm), liquid properties (μ_L= 0.018 - 0.1 kg/m.s; σ = 0.061 - 0.8 N/m) on C_L and proposed a correlation for C_L as a linear function of Eo_H based on the horizontal bubble diameter. Unfortunately, this correlation was found to be valid for only higher liquid viscosities ($4 \le Eo_H \le 8$, $-3.6 \le \log Mo \le -5.7$), and could not be applied to less viscous fluids such as water. Rabha and Buwa [13] investigated the effect of bubble size/shape and the effect of neighbouring bubbles on the magnitude and direction of lift force. Further, they investigated the lift force acting on single bubbles rising in low viscosity fluids (corresponding to the wobbling regime, $1.1 \le Eo \le 8.72$, $-10.6 \le \log Mo \le -14.6$). They observed that unlike the steady lateral migration of single bubbles with single characteristics value of C_L (+ve or –ve depending on d_B) for viscous systems, the bubbles were found to oscillate around the center line and the instantaneous C_L was found to fluctuate in both +ve and –ve directions.

Van Helden et. al. [14] carried out experimental measurementson the bubble detachment near a plane in a vertical rectangular channel. The authors used two different fluids for bubble generation namely (1) vapour bubble (2) nitrogen bubble. The authors observed that the steam bubbles traveled towards

the bulk liquid (away from the wall) while the nitrogen bubbles travel parallel to the wall in the bulk liquid. The authors deduced two notable conclusions from their study namely (i) the bubble detachment depended significantly on the bulk density of continuous phase ith radius of bubble detachment decreasing with increase in bulk density (ii) theoretically derived expressions for lift coefficients (Auton[10]) were unable to predict lift coefficients for bubbles moving near the wall

Felton and Loth [15] carried out experimental investigations on dynamics of spherical air bubbles in water on the basis of bubble velocities in terms of Stokes number (ratio of characteristic time scale of bubble to that of flow which depends on fluid velocity and length scale (example boundary layer thickness)). The authors concluded that for high Stokes number, the bubbles got accumulated along the wall and a Gaussian distribution of bubbles across horizontal plane for low Stokes number. Further, three different types of bubble trajectories namely sliding bubbles, bouncing bubbles and free-dispersion bubbles were also observed.

Tomiyama et. al. [6] experimentally investigated the estimation of transverse lift force acting on single bubbles in simple shear flows on glycerol–water system. The authors observed that the net transverse lift coefficient for small bubbles is a function of the bubble Reynolds number. However, for larger bubbles the lift coefficient was found to be a function of modified Eötvös number. The authors estimated expressions for critical bubble diameter as a function of lift coefficient.

Rabha and Buwa [13] carried out numerical investigations on the effects of the size of bubbles, the number of neighbouring bubbles and the shape of bubbles on the magnitude and direction of the lift force. The authors found that the direction of lift coefficient depended on the viscosity of the system with oscillatory behaviour in low viscosity systems and steady behaviour in high viscosity systems. Further, as per the authors, the magnitudes of lift coefficient depended on bubble-bubble interactions. An increase in the magnitude was observed when bubble-bubble interactions were higher irrespective of the properties of the continuous/stagnant liquid phase.

Aoyama et. al. [16] experimentally investigated the effects of the bubble shape, the liquid velocity gradient and the fluid properties on the magnitude of lift coefficientfor two different bubble shapes namely spherical and ellipsoidal. The authors observed that the lift coefficient of spherical bubbles at low Reynolds number increases with an increase in Shear Rate while it decreases with an increase in Reynolds number.

Hessenkemper et. al. [17] experimentally investigated the effects of inorganic surfactant (NaCl), on the lift force for single bubbles. The authors concluded that the lift coefficients for small bubbles got affected by the addition of inorganic surfactantwhile those for larger bubbles got affected only upto a certain critical concentration (0.1 mg·l-1), after which it weakly affected the C_L values.

Hidman et al.[18] numerically investigated two different parameters: (1) the lift force acting on a freely deformable bubble rising in a linear shear flow (2) the lift force scaling with the undisturbed shear rate for different lift force mechanisms. The authors investigated the lift coefficient as a function of various non-dimensionelss numbers Eo, Ga, Sr and Re. The authors observed that for highly viscous flows and for significant bubble deformations, the Shear Rate becomes an important study parameter for modeling of the lift force coefficient.

Table 1 Details of various geometric and operating parameters for lift coefficient measurement

Reference	**Geometric and operating parameters**					**Channel Details**	**Details of Dimensionless Number(s)**	**Lift Coefficient Range**
	Continuous phase	**Dispersed phase**						
	Medium	**Flow Velocity (ms^{-1})**	**Medium**	**Shape**	**Bubble size (mm)**			
Van Helden, Van Der Geld [14]	Water	Steam: 0.16 – 0.85 Nitrogen: 0 – 0.98	Steam Nitrogen	Spherical	Steam: 0.34 – 1.104 Nitrogen: 0.62 – 3.348	Vertical Rectangular channel	11000 ≤ Re ≤ 58000	–

Felton and Loth [15]	Water	0.4 – 0.9	Air	Spherical	0.37 – 1.2	Vertical Rectangular channel	$13000 \leq \mathrm{Re} \leq 20000$	$0.001 \leq C_L \leq 0.5$
Tomiyama, Tamai [6]	Glycerol – water solution	0 – 0.27	Air	Spherical	2.84 – 5.54	Vertical Rectangular channel	$1.39 \leq \mathrm{Eo} \leq 5.74$ $-5.5 \leq \log_{10} \mathrm{M} \leq -2.8$	–
Rabha and Buwa [13]	Water Glycerol – water solution	–	Air	Spherical Ellipsoidal	3.52 – 8	Vertical Rectangular channel with constant shear	$1.1 \leq \mathrm{Eo} \leq 11.68$ $-5.5 \leq \log_{10} \mathrm{M} \leq -2.8$	$-6 \leq C_L \leq 10$
Masood and Delgado [19]	Water	0.0049	Air	–	–	Vertical Rectangular channel	–	–
Aoyama, Hayashi [16]	Glycerol – water solution	–	Air	Ellipsoidal	0.69 – 5.34	Vertical Rectangular channel with constant shear	$0.7 \leq \mathrm{Re} \leq 119$ $-6.6 \leq \log_{10} \mathrm{M} \leq -3.2$	$-0.28 \leq C_L \leq 1.49$
Hessenkemper, Ziegenhein [17]	Water – salt (NaCl) solution	0.003 – 0.08	Air	Ellipsoidal	2.2 – 6.5	Vertical Rectangular channel	$1.13 \leq \mathrm{Eo} \leq 3.57$	$-0.28 \leq C_L \leq 0.8$
Muniz and Sommerfeld [20]	Water	–	Air	Ellipsoidal	2.55 – 3.31	Cylindrical geometry	$0 \leq \mathrm{Eo} \leq 3$ $0 \leq \mathrm{Re} \leq 1400$	$0.4 \leq C_L \leq 0.8$
Hidman, Ström [18]	Water	–	Air	Ellipsoidal	–	Vertical Rectangular channel with constant shear	$0.1 \leq \mathrm{Eo} \leq 10$ $0.77 \leq \mathrm{Re} \leq 265$	$-7.47 \leq C_L \leq 1.46$
Subburaj, Tang [21]	Water	–	Air	Spherical	4	Vertical Rectangular channel	–	$0 \leq C_L \leq 0.6$

Subburaj et al. [21] carried out numerical studies for lift coefficient evaluation considering for the variation in the bubble spreading mechanism for different aspect ratios of the column with homogenous and heterogenous bubbly flows. The authors observed that the local gas fraction and the Shear Rate were simultaneously correlated with the lift coefficient. The authors also observed that decreasing decrease in the gas fraction reduced the magnitude of the lift force, while a bubble in the proximity of another bubble experienced higher lift. This is a contradictory observation as compared to[22].

Table 2 Correlations for Lift coefficients

Reference	Correlation
Tomiyama, Tamai [6]	$C_L = \begin{cases} \min[0.288\tanh(0.121\mathrm{Re}), f(\mathrm{Eo})], & \mathrm{Eo} < 4 \\ f(\mathrm{Eo}), & 4 \leq \mathrm{Eo} \leq 10.7 \end{cases}$ $f(\mathrm{Eo}) = 0.00105\mathrm{Eo}^3 - 0.0159\mathrm{Eo}^2 - 0.0204\mathrm{Eo} + 0.474$
Bothe, Schmidtke [23]	$C_L = \begin{cases} 0.29, & \mathrm{Eo} < 4 \\ f(\mathrm{Eo}), & 4 \leq \mathrm{Eo} \leq 10.7 \end{cases}$ $f(\mathrm{Eo}) = 0.00105\mathrm{Eo}^3 - 0.0159\mathrm{Eo}^2 - 0.0204\mathrm{Eo} + 0.474$
Muniz and Sommerfeld [20]	$C_L = \begin{cases} \min[0.288\tanh(0.121\mathrm{Re}), f(\mathrm{Eo})], & \mathrm{Eo} \leq 4 \\ f(\mathrm{Eo}), & 4 < \mathrm{Eo} \leq 10 \\ -0.288, & 10 < \mathrm{Eo} \end{cases}$

	$f(Eo) = 0.00105Eo^3 - 0.0159Eo^2 - 0.0204Eo + 0.474$
Hessenkemper, Ziegenhein [24]	$C_L = C_L' - f(Eo')$ $C_L' = \sqrt{\left(\frac{13.5}{\left(1 + 0.2\frac{Re}{Sr}\right)^{3/2}}{\pi^2\sqrt{ReSr}}\right)^2 + \left(\frac{1}{2}\frac{Re + 16}{Re + 29}\right)^2}$ $f(Eo') = \frac{\ln(1 + \exp(-12G))}{12}$ $G = 0.11\frac{\ln(1 + \exp[4(Eo - 5.6)])}{4} - 0.14(Eo - 5.2) - 0.44$
Subburaj, Tang [21]	$C_L = \begin{cases} 0.6 - 0.89(y/H), & (y/H) \le 0.278 \\ 1.4 - 4.44(y/H), & 0.278 < (y/H) < 0.38 \\ 0, & (y/H) \ge 0.38 \end{cases}$

1.2. Objective of present work

The review of published literature shows that most of the work carried for evaluation of lift coefficients is for air-water systems and those for vapor-liquid systems are not available. Further, the lift forces are evaluated for higher viscous systems for different ranges of Eötvös and Morton numbers. The generation of bubbles have been mainly though single spargers and the shear has been induced using mechanical systems like moving belt etc.

In the present work, a system different than the conventional GL system has been chosen. Here, the shear induced was due to the thermal boundary layer on a vertical wall maintained at constant heat flux. Further, the bubbles are generated from the nucleation sites created on the wall. The three main objectives of the evaluation of lift coefficient are 1. Determination of bubble size as a function of Rayleigh number 2. Determination of empirical correlation for determination of lift coefficient as a function of dimensionless numbers 3. Building a robust CFD model capable of predicting lift coefficients for vapor-liquid systems.

2. Experimental and CFD details

For experimental measurements of the bubble size, shape and trajectory a High speed Camera (Photron Fastcam 10KC, Resolution 512×480 pixels) was used. The images were analyzed by means of MATLAB software. All the instruments were calibrated before testing. For CFD simulations, a bubble on the vertical wall (represented as line in a 2D axisymmetric geometry) has been patched and the VOF multiphase model has been implemented with turbulence given by SST k-ω model. The governing equations of momentum, energy, turbulence and VOF are same as used by [25] in Ansys Fluent 14. The spatial derivatives for the governing equations of momentum were discretized using second order upwind scheme, energy was discretized using first order upwind scheme while turbulence were discretized using the QUICK scheme [26]. A first-order implicit method was used for the discretization of the temporal derivatives. The Pressure Implicit with Splitting of Operator (PISO) algorithm [27] was used for the pressure-velocity coupling in the momentum equation. The procedure for transient simulation is as follows:

A single phase simulation has been carried out for the first 20 seconds when the first bubble departs as observed in the experiments of Ra = 3.76E+13. A bubble of suitable diameter as calculated from the measured bubble size by high speed camera has been patched as an initial condition and two phase simulation using VOF method has been used. A no slip boundary condition was used at the stationary wall. The effect of column depth (D) on the bubble rise velocity (V_R) and lift coefficient (C_L) has been studied. The time step was kept as $1x10^{-6}$ s for all simulations.

3. Results and discussion

3.1. Effect of Ra number on bubble size

In the present investigation of vapor-liquid flows it has been observed that bubble size during inception and movement is a function of Rayleigh number. It was also observed that during the 55% of the total time the vapor bubble nearly retains its size with negligible condensation since it remains in the region of thermal boundary layer for all Ra numbers except the highest Ra number considered. For the highest Ra number, the time duration is 38% of the overall time after it starts condensing. Hence, the lift force calculation has been done considering the above aspects during bubble dynamics and the time of condensation and change in bubble size as a function of both spatial and temporal variation has not been considered. Figure 1 shows the bubble size as a function of Ra number for the Ra number range 8 x 10^{12} < Ra < 3.76 x 10^{13}. As the Ra increases, the bubble size linearly increases till Ra = 1.8E+13. The bubble size variation tapers down with further increase in Ra number. The experimental and simulation predictions show a good agreement with a deviation of 4-7% with CFD simulations underpredicting the experimental data. This is attributed to the two dimensional nature of simulations. An empirical correlation for the bubble size as a function of Ra is given below:

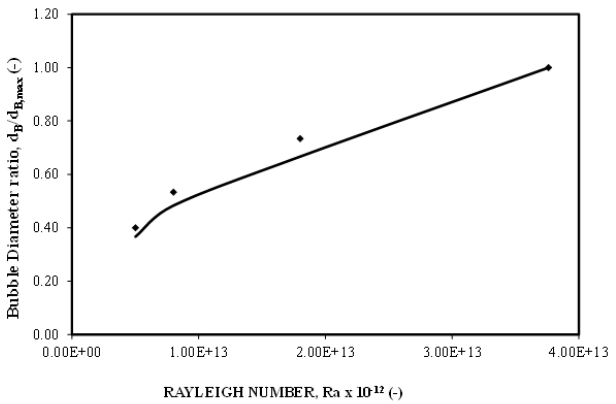


Figure 1 Variation of bubble diameter ratio with Rayleigh number (Ra). Symbols denote experimental data while line denotes CFD predicted data

$$d_B/d_{B,max} = 0.203Ra^{0.443} \qquad 8 \text{ x } 10^{12} < \text{Ra} < 3.76 \text{ x } 10^{13};\ d_{B,\,max} = 3 \text{ mm} \qquad (3)$$

3.2. Evaluation of lift coefficient

In the present section we present the results of the lift force exerted on the bubble when a specific amount of heat input is provided are reported. The present simulations have been provided after 60 seconds of run when the first bubble appears as per experimental observations. Rayleigh number (Ra) has been varied in the range 7.9e+12 < Ra < 3.76e+13. The lift force has been calculated using the expression below

$$C_L = \frac{g}{\omega}\left(\frac{V_{Rx}}{V_{Rx}^2 + V_{Ry}^2}\right) \qquad (4)$$

The lift coefficient calculations (both experimental and numerical cases) have been carried out according to Eq. 4. The major difference between air bubbles in water in wall regime is the existence of thermal boundary layer and the intrusion layer that forms at the top surface. Further, since the bubble motion starts from the wall and the inception of the bubble, its formation are dependent on nucleation site, it is different than single bubble motion from the center. For the present case the bubble motion has been explained in our previous work (Ganguli et al. [25]). The lateral migration, length or distance from the wall depended on the relative velocity gradient. A positive lift coefficient has been observed for the time period considered and for all the heat fluxes under consideration. In the present case of subcooled pool boiling the bubbles should be having positive values in the wall regime. Although positive values are observed both in simulation and experiments and the bubbles move in rectilinear motion at low heat fluxes, the important observation is that after a certain vertical distance the bubbles depart away from the wall. This suggests that there is a good balance between the buoyancy, surface tension and viscous force upto a certain vertical distance after which the buoyancy

forces overcome the viscous resistance and move away from the wall. Thus, similar to Tomiyama et al. [6] we can predict that the bubble is in the wall regime depending on the shear due to the hydrodynamic and thermal boundary layers.

Figure 1A shows the lift coefficient variation with Reynolds number for Ra number range 8 x 10^{12} < Ra < 3.76 x 10^{13}. The simulation results agree reasonably well with the experimental measurements with 6% under-predictions which is attributed to the 2D simulations carried out. An empirical correlation for lift coefficient as a function of Re has been given below:

$$C_L = -3E-05Re^3 + 1E-03Re^2 - 0.00026Re - 0.0034$$

$\log_{10}$ M = -10.6; 0.2 < Eo < 1.3; 8 x 10^{12} < Ra < 3.76 x 10^{13} (5)

Interestingly, the highest values lift coefficient (C_L) are around 0.1 for a bubble size of 3 mm. Further, the Reynolds numbers (based on the resultant velocity and bubble diameters obtained) are in the range of 5 < Re < 30 which is similar to the Reynolds numbers considered by Tomiyama et al. [6] but due to the shear mechanism and different Morton number the lift coefficients magnitudes are much lower than the ones observed by Tomiyama et al. [6]. By the lower values of lift force it may be attributed to several factors like the mass transfer taking place between the liquid and vapor bubble, the vapor shear rate and the unsteady movement of the bubble (since the lift coefficient is averaged over time and the vapor bubble after detaching has lower lift coefficients than when it has traveled a certain distance) hence giving us values which are one third of the values if C_L in published literature.

Figure 2B shows the variation of C_L with respect of Eö. The tapering of C_L values after Eö value of 0.6 can be observed. This is a significant result considering the fact that the C_L values for this range have not been reported for such lower range of Eö numbers at the specified Morton number and the range of Ra numbers considered. The lower magnitude of lift force and positive sign is attributed to the explanation provided previously in Figure 2. However, this aspect needs to be looked in more detail and has not been included presently. An empirical correlation for lift coefficient as a function of Eö has been given below:

$$C_L = -0.1401E\ddot{o}^2 + 0.2803E\ddot{o} - 0.0373$$

$\log_{10}$ M = -10.6; 0.2 < Eo < 1.3; 8 x 10^{12} < Ra < 3.76 x 10^{13} *(6)*

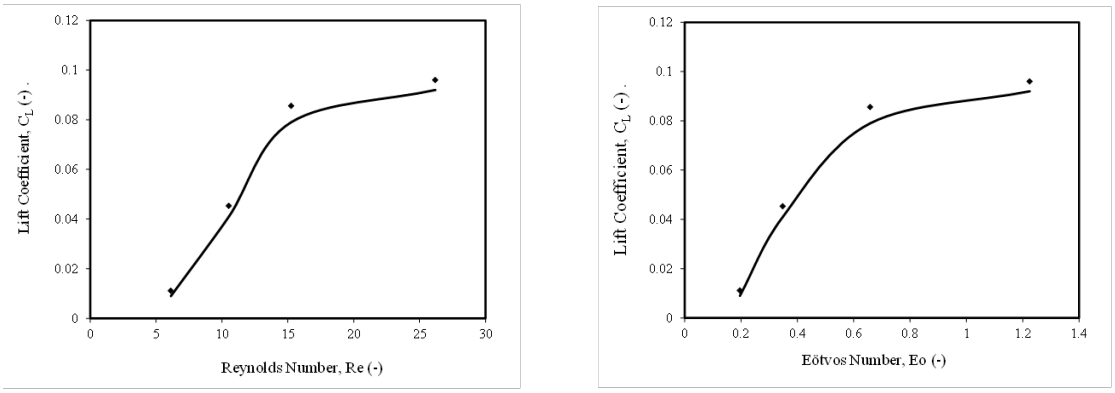


Figure 2 Variation of lift coefficient with (A) Reynolds number. (B) Eötvös number (Eo). Symbols denote experimental data while line denotes CFD predicted data

References

1. Magnus, G., *Ueber die Abweichung der Geschosse, und: Ueber eine auffallende Erscheinung bei rotirenden Körpern*. Annalen der physik, 1853. **164**(1): p. 1-29.

2. Svendsen, H., H. Jakobsen, and R. Torvik, *Local flow structures in internal loop and bubble column reactors*. Chemical Engineering Science, 1992. **47**(13-14): p. 3297-3304.
3. Tzeng, J.W., R. Chen, and L.S. Fan, *Visualization of flow characteristics in a 2-D bubble column and three-phase fluidized bed*. AIChE journal, 1993. **39**(5): p. 733-744.
4. Saffman, P., *The Lift on a Small Sphere in a Slow*. 1968.
5. Thomas, N., et al. *Entrapment and transport of bubbles by transient large eddies in multiphase turbulent shear flows*. in *International conference on the physical modelling of multi-phase flow*. 1983.
6. Tomiyama, A., et al., *Transverse migration of single bubbles in simple shear flows*. Chemical Engineering Science, 2002. **57**(11): p. 1849-1858.
7. Kariyasaki, A. *Behavior of a single gas bubble in a liquid flow with a linear velocity profile*. in *Proceedings of the 1987 ASME-JSME Thermal Engineering Joint Conference, ASME New York*. 1987.
8. Rafique, M., P. Chen, and M. Duduković, *Computational modeling of gas-liquid flow in bubble columns*. Reviews in Chemical Engineering, 2004. **20**(3-4): p. 225-375.
9. Ẑun, I., *The transverse migration of bubbles influenced by walls in vertical bubbly flow*. International Journal of Multiphase Flow, 1980. **6**(6): p. 583-588.
10. Auton, T., *The lift force on a spherical body in a rotational flow*. Journal of fluid Mechanics, 1987. **183**: p. 199-218.
11. Drew, D. and R. Lahey Jr, *The virtual mass and lift force on a sphere in rotating and straining inviscid flow*. International Journal of Multiphase Flow, 1987. **13**(1): p. 113-121.
12. Bothe, D., M. Schmidtke, and H. Warnecke, *VOF-simulation of the rise behavior of single air bubbles in linear shear flows*. Chemical Engineering and Technology, 2006. **29**: p. 1048-1053.
13. Rabha, S.S. and V.V. Buwa, *Volume-of-fluid (VOF) simulations of rise of single/multiple bubbles in sheared liquids*. Chemical Engineering Science, 2010. **65**(1): p. 527-537.
14. Van Helden, W., C. Van Der Geld, and P. Boot, *Forces on bubbles growing and detaching in flow along a vertical wall*. International journal of heat and mass transfer, 1995. **38**(11): p. 2075-2088.
15. Felton, K. and E. Loth, *Spherical bubble motion in a turbulent boundary layer*. Physics of Fluids, 2001. **13**(9): p. 2564-2577.
16. Aoyama, S., et al., *Lift force acting on single bubbles in linear shear flows*. International Journal of Multiphase Flow, 2017. **96**: p. 113-122.
17. Hessenkemper, H., T. Ziegenhein, and D. Lucas, *Contamination effects on the lift force of ellipsoidal air bubbles rising in saline water solutions*. Chemical Engineering Journal, 2020. **386**: p. 121589.
18. Hidman, N., et al., *The lift force on deformable and freely moving bubbles in linear shear flows*. Journal of Fluid Mechanics, 2022. **952**: p. A34.
19. Masood, R. and A. Delgado, *Numerical investigation of the interphase forces and turbulence closure in 3D square bubble columns*. Chemical Engineering Science, 2014. **108**: p. 154-168.
20. Muniz, M. and M. Sommerfeld, *On the force competition in bubble columns: A numerical study*. International Journal of Multiphase Flow, 2020. **128**: p. 103256.
21. Subburaj, R., Y. Tang, and N.G. Deen, *An improved lift model for Euler-Lagrange simulations of bubble columns*. Chemical Engineering Science, 2023. **265**: p. 118182.
22. Hallez, Y. and D. Legendre, *Interaction between two spherical bubbles rising in a viscous liquid*. Journal of fluid mechanics, 2011. **673**: p. 406-431.
23. Bothe, D., M. Schmidtke, and H.J. Warnecke, *VOF-simulation of the lift force for single bubbles in a simple shear flow*. Chemical Engineering & Technology: Industrial Chemistry-Plant Equipment-Process Engineering-Biotechnology, 2006. **29**(9): p. 1048-1053.
24. Hessenkemper, H., et al., *Lift force coefficient of ellipsoidal single bubbles in water*. International Journal of Multiphase Flow, 2021. **138**: p. 103587.

25. Ganguli, A.A., A.B. Pandit, and J.B. Joshi, *Bubble dynamics of a single condensing vapor bubble from vertically heated wall in subcooled pool boiling system: experimental measurements and CFD simulations*. International Journal of Chemical Engineering, 2012. **2012**.
26. Leonard, B.P., *A stable and accurate convective modelling procedure based on quadratic upstream interpolation*. Computer methods in applied mechanics and engineering, 1979. **19**(1): p. 59-98.
27. Issa, R.I., *Solution of the implicitly discretised fluid flow equations by operator-splitting*. Journal of computational physics, 1986. **62**(1): p. 40-65.